

Security in Computer Networks

Multilateral Security in and by Distributed Systems

Transparencies for the Lecture:

Security and Cryptography I

(Version 2022/11/10)

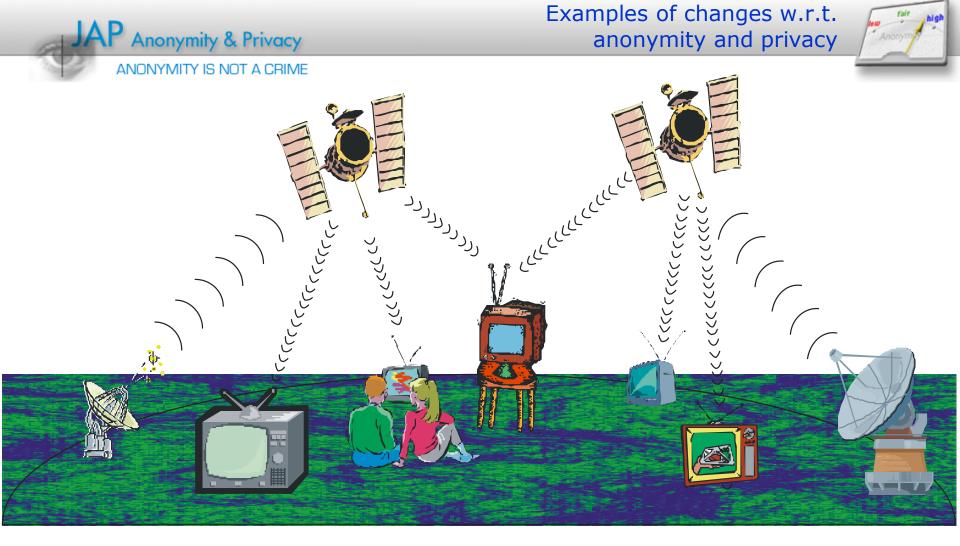
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Lectures Security and Cryptography I, II	Staff Köpsell Köpsell	SWS 2/2 2/0
Application Security Cryptography and -analysis	Köpsell Franz	2/0 2/1
Information & Coding Theory	Franz	2/1
Data Security and Cryptography	Köpsell	0/4
Security Lab	Köpsell	<mark>2/2</mark>
Computers and Society	Köpsell	2/0
Seminar: Privacy and Security	Byrenheid et al.	2/0
Seminar: Security in Computer Systems	Köpsell	2
Introduction to Data Protection Law	Wagner	2/0
Resilient Networking	Strufe	2/2
Privacy Enhancing Technologies	Strufe	3/1

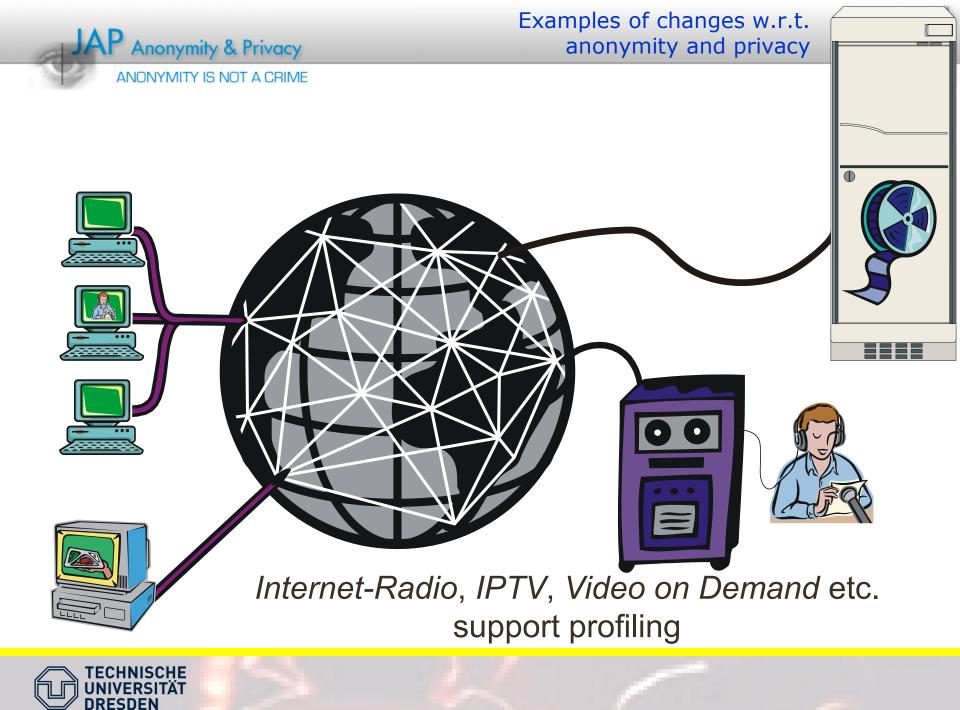


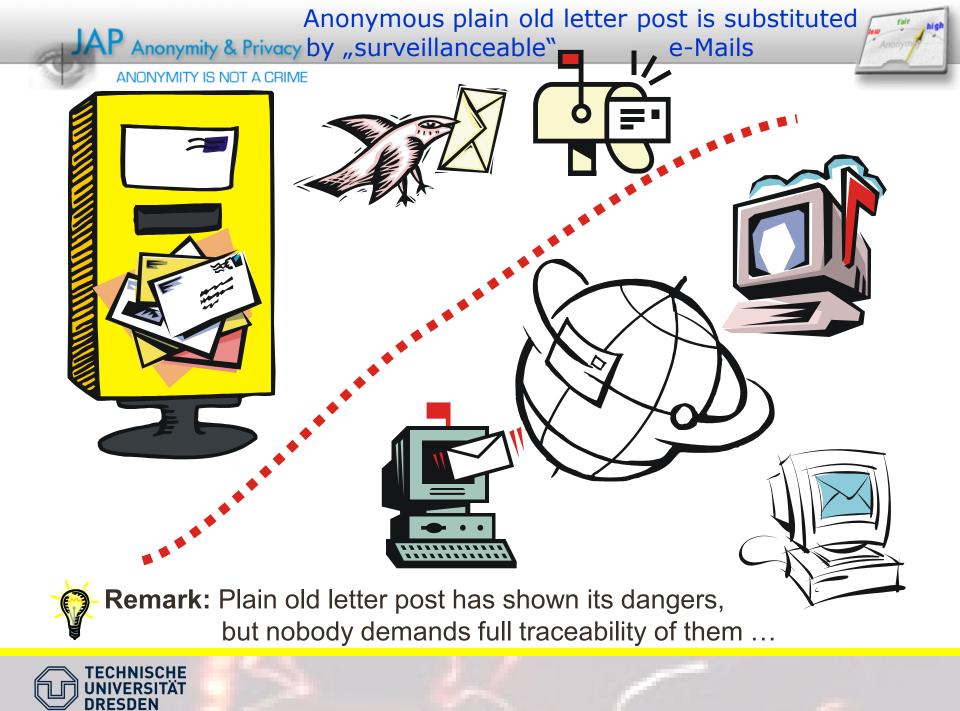
- Multilateral security, in particular security by distributed systems
- Privacy Enhancing Technologies (PETs)
- Cryptography
- Physical Layer Security
- Information- and coding theory
- Security & Privacy
 - in Vehicular Networks (Connected Driving)
 - for IoT & Cyberphysical Systems
 - industrial communication
 - focused on humans: social engineering, transparency
- Context-aware, Adaptive & Smart Security Solutions



Broadcast allows recipient anonymity — it is not detectable who is interested in which programme and information







The massmedia "newspaper" will be personalised by means of Web, elektronic paper and print on demand



ANONYMITY IS NOT A CRIME

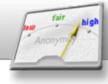
AP Anonymity & Privacy

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The Bystander-Challenge in Smart IoT Worlds...



ANONYMITY IS NOT A CRIME

AP Anonymity & Privacy

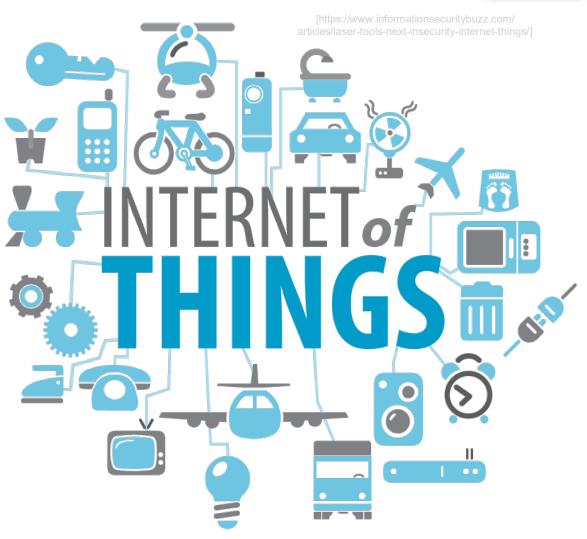
ubiquitous sensing

undermines privacy in everybody's everyday life

- cameras, microphones, Lidar, Radar, ...
- examples: connected driving, Smart Cities, e-Health, ...

bystanders are affected

- awareness and control for uninvolved persons
- example: voice assistants
- # reasoning about linkability and privacy in complex and realistic systems







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- State-of-the-Art:
 - manual, highly application dependent engineering of security/privacy solutions by security experts
 - → costly
 - does not scale
- Solution:
 - automation of the security development, deployment and operation processes
- Ingredients:
 - Reasoning about Situation: Context & Context Awareness
 - Measuring/Specifying Security: Quality of Security
 - Decisions: AI & ML (or something better)
- Result: Disruption for Digitalisation
- Minor issue: no clue how to achieve it...



Science shall clarify *How something is.*

But additionally, and even more important *Why it is such*

or

How could it be

(and sometimes, how should it be).

"**Eternal truths**" (i.e., knowledge of long-lasting relevance) should make up more than 90% of the teaching and learning effort at universities.

General Aims of Education in IT-security (sorted by priorities)

12

- 1. Education to **honesty** and a **realistic self-assessment**
- 2. Encouraging realistic **assessment of others**, e.g., other persons, companies, organizations
- 3. Ability to gather security and data protection requirements
 - Realistic protection goals
 - Realistic attacker models / trust models



General Aims of Education in IT-security (sorted by priorities)

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 - Realistic protection goals
 - Realistic attacker models / trust models
- 4. Validation and verification, including their practical and theoretical limits
- 5. Security and data protection **mechanisms**
 - Know and understand as well as
 - Being able to develop

In short: Honest IT security experts with their own opinion and personal strength.

- Education to honesty and a realistic self-assessment
 As teacher, you should make clear
 - your strengths and weaknesses as well as
 - your limits.

Oral examinations:

- Wrong answers are much worse than "I do not know".
- Possibility to explicitly exclude some topics at the very start of the examination (if less than 25% of each course, no downgrading of the mark given).
- Offer to start with a favourite topic of the examined person.
- Examining into depth until knowledge ends be it of the examiner or of the examined person.

- 1. Education to **honesty** and a **realistic self-assessment**
- 2. Encouraging realistic **assessment of others**, e.g., other persons, companies, organizations

Tell, discuss, and evaluate case examples and anecdotes taken from first hand experience.

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- 1. Education to **honesty** and a **realistic self-assessment**
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 - Realistic attacker models / trust models

Tell, discuss, and evaluate case examples (and anecdotes) taken from first hand experience.

Students should develop scenarios and discuss them with each other.

- 1. Education to **honesty** and a **realistic self-assessment**
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- 3. Ability to gather security and data protection requirements
 - Realistic protection goals
 - Realistic attacker models / trust models
- 4. Validation and verification, including their practical and theoretical limits

Work on case examples and discuss them.

Anecdotes!

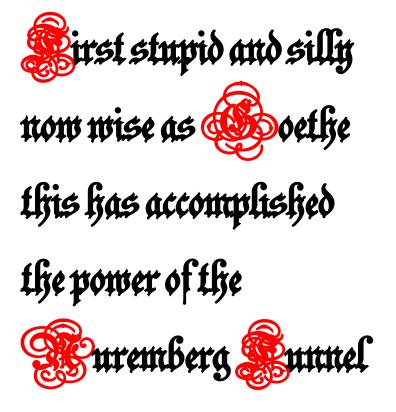
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Whatever students can discover by themselves in exercises should not be taught in lectures.



...but no this way!





Nuremberg Funnel (German: Nürnberger Trichter) Postcard from around 1940



Offers by the Chair of Privacy and Data Security

- Interactions between IT-systems and society, e.g., conflicting legitimate interests of different actors, privacy problems, vulnerabilities ...
- Understand fundamental security weaknesses of today's ITsystems
- Understand what Multilateral security means, how it can be characterized and achieved
- Deepened knowledge of the important tools to enable security in distributed systems based on cryptography
- Deepened knowledge in error-free transmission and playback
- Basic knowledge in fault tolerance
- Considerations in building systems: expenses vs. performance vs. security
- Basic knowledge in the relevant legal regulations



- Deepened knowledge security in operating systems
- Verification of OS kernels
- Deepened knowledge in fault tolerance
- Deepened knowledge in trusted execution environments

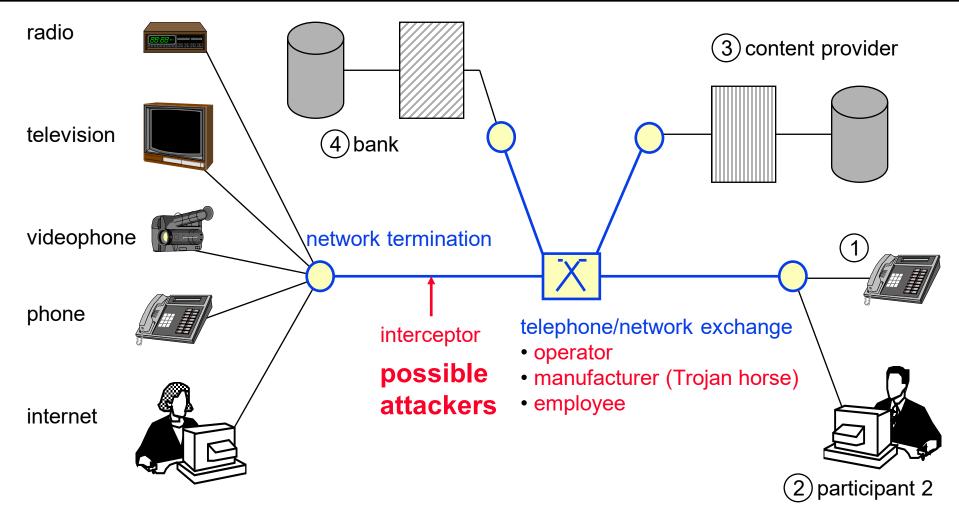


1 Introduction

- 1.1 What are computer networks (open distributed systems) ?
- 1.2 What does security mean?
 - 1.2.1 What has to be protected?
 - 1.2.2 Protection against whom?
 - 1.2.3 How can you provide for security?
 - 1.2.4 Protection measures an overview
 - 1.2.5 Attacker model
- 1.3 What does security in computer networks mean?
- 2 Security in single computers and its limits
 - 2.1 Physical security
 - 2.1.1 What can you expect at best?
 - 2.1.2 Development of protection measures
 - 2.1.3 A negative example: Smart cards
 - 2.1.4 Reasonable assumptions on physical security
 - 2.2 Protecting isolated computers against unauthorized access and computer viruses
 - 2.2.1 Identification
 - 2.2.2 Admission control
 - 2.2.3 Access control
 - 2.2.4 Limitation of the threat "computer virus" to "transitive Trojan horse"
 - 2.2.5 Remaining problems
- 3 Cryptographic basics



Part of a Computer Network



example. (5) monitoring of patients,(6) transmission of moving pictures during an operation

Why are legal provisions (for security and data protection) not enough ?



- 1833 First electromagnetic telegraph
- 1858 First cable link between Europe and North America
- 1876 Phone operating across a 8,5 km long test track
- 1881 First regional switched phone network
- 1900 Beginning of wireless telegraphy
- 1906 Introduction of **subscriber trunk dialing** in Germany, realized by two-motion selector, i.e., the first fully automatic telephone exchange through electro-mechanics
- 1928 Introduction of a telephone service Germany-USA, via radio
- 1949 First working von-Neumann-computer
- 1956 First transatlantic telephone line
- 1960 First communications satellite
- 1967 The datex network of the German Post starts operation,

i.e., the first communication network realized particularly for computer communication (computer network of the first type). The transmission was digital, the switching by computers (computer network of the second type).

1977 Introduction of the electronic dialing system **(EWS)** for telephone through the German Post, i.e., the first telephone switch implemented by computer (computer network of the second type), but still analogue transmission



1981 First personal computer (PC) of the computer family (**IBM PC**), which is widely used in private households 1982 investments in phone network transmission systems are increasingly in digital technology 1985 Investments in telephone switches are increasingly in computer-controlled technology. Now transmission is no longer analogue, but **digital signals are switched and transmitted** (completed 1998 in Germany) 1988 Start-up of the **ISDN** (Integrated Services Digital Network) 1989 First pocket PC: Atari Portfolio; so the computer gets personal in the narrower sense and mobile 1993 **Cellular phone networks** are becoming a mass communication service 1994 www commercialization of the Internet 2000 **WAP-capable mobiles** for 77 € without mandatory subscription to services 2003 with IEEE 802.11b, WLAN (Wireless Local Area Network) and Bluetooth **WPAN** (Wireless Personal Area Network) find mass distribution 2004 **UMTS** starts in Germany

2005 VoIP (Voice over IP) is becoming a mass communication service

- 2007 first generation iPhone
- 2012 LTE with up to 300 MBit/s



computers interconnected by **communication network** = **computer network** (of the first type)

distributed system

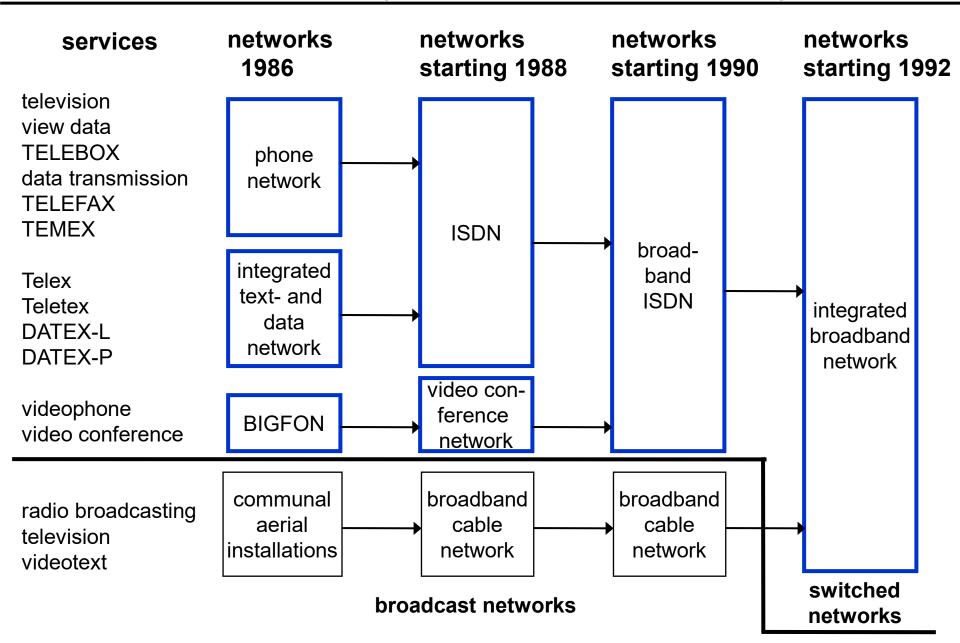
spatial control and implementation structure

open system \neq **public** system \neq **open source** system

service integrated system

digital system

Development of the fixed communication networks of the ²⁸ German Post (Roadmap of approx. 1982)





2)+3)

<u>threa</u>	example: medical information	ation system	protection goals:
,	nauthorized access to information mputer company receives medical files		confidentiality
· · ·	authorized modification of information detected change of medication	on ≥ total ≺	
in de no cl	co horized withholding of hation or resources ed failure of system sification, but pragmatically use unauthorized modification of a progra		_ availability for authorized users
1)	cannot be detected, but can be pro	evented;	cannot be reversed

cannot be prevented, but can be detected;

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can be reversed



threats:	example: medical inform	nation system	protection goals:
,	d access to information pany receives medical files		c onfidentiality
	d modification of information ange of medication	≥ total ≺	$ \begin{cases} integrity \\ \cong partial correctness \end{cases} $
detected failure no classificat	or resources	correctness	availability for authorized users
1) cannot	be detected, but can be p	revented;	cannot be reversed

2)+3) cannot be detected, but can be prevented;2)+3) cannot be prevented, but can be detected;

cannot be reversed can be reversed



confidentiality

Only authorized users get the information.

integrity

Information are correct, complete, and current or this is detectably not the case.

availability

Information and resources are accessible where and when the authorized user needs them.

- subsume: data, programs, hardware structure
- it has to be clear, who is authorized to do what in which situation
- it can only refer to the inside of a system



	Content	Circumstances
Prevent the unintended	Confidentiality Hiding	Anonymity Unobservability
Achieve the intended	Integrity	Accountability
	Availability	Reachability Legal Enforceability



Confidentiality ensures that nobody apart from the communicants can discover the content of the communication.

Hiding ensures the confidentiality of the transfer of confidential user data. This means that nobody apart from the communicants can discover the existence of confidential communication.

Anonymity ensures that a user can use a resource or service without disclosing his/her identity. Not even the communicants can discover the identity of each other.

Unobservability ensures that a user can use a resource or service without others being able to observe that the resource or service is being used. Parties not involved in the communication can observe neither the sending nor the receiving of messages.

Unlinkability ensures that an attacker cannot sufficiently distinguish whether two or more items of interest (subjects, messages, actions, ...) are related or not.

Integrity ensures that modifications of communicated content (including the sender's name, if one is provided) are detected by the recipient(s).

Accountability ensures that sender and recipients of information cannot successfully deny having sent or received the information. This means that communication takes place in a provable way.

Availability ensures that communicated messages are available when the user wants to use them.

Reachability ensures that a peer entity (user, machine, etc.) either can or cannot be contacted depending on user interests.

Legal enforceability ensures that a user can be held liable to fulfill his/her legal responsibilities within a reasonable period of time.

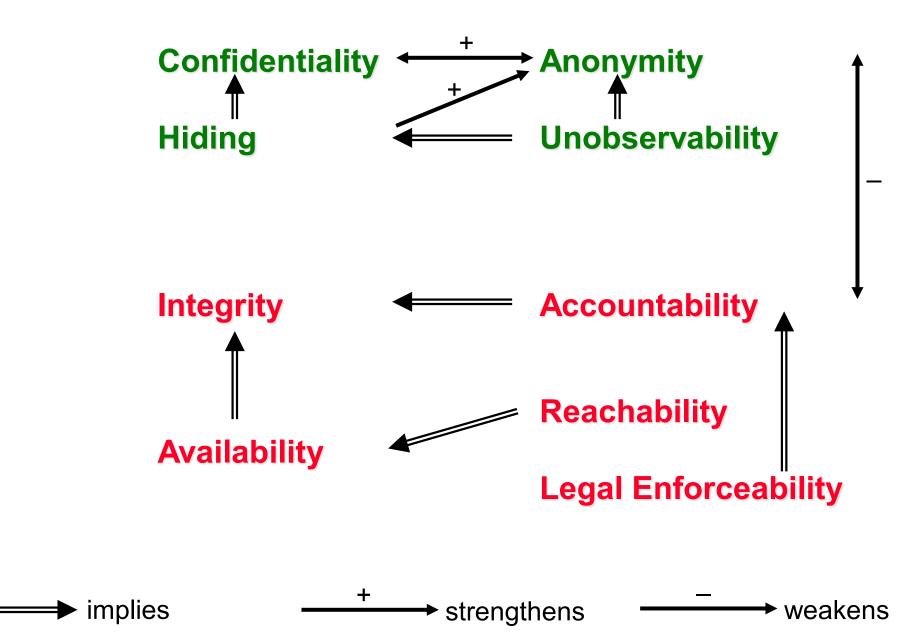


Additional Data Protection Goals: Definitions (Rost/Pfitzmann 2009)

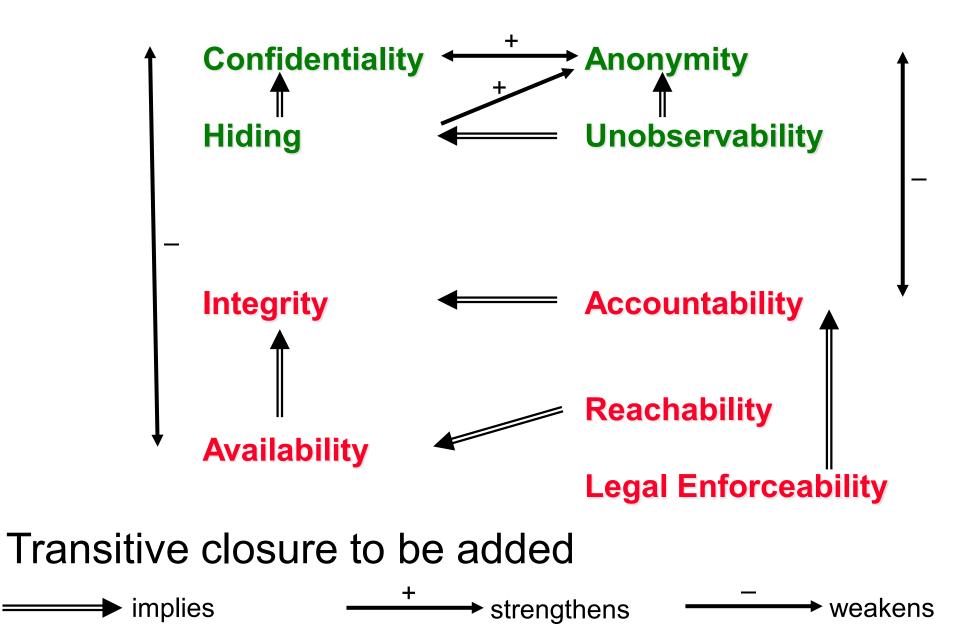
Transparency ensures that the data collection and data processing operations can be planned, reproduced, checked and evaluated with reasonable efforts.

Intervenability ensures that the user is able to exercise his or her entitled rights within a reasonable period of time.



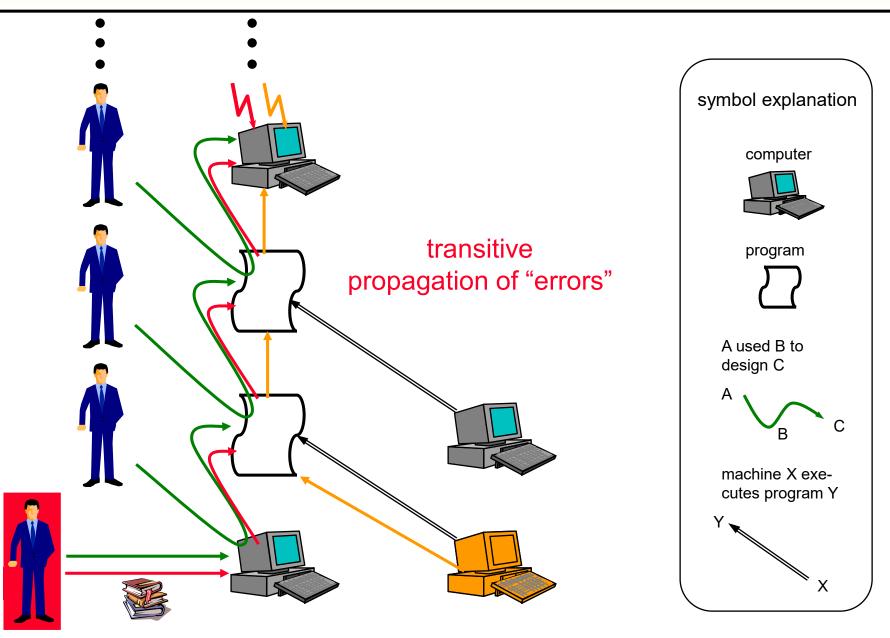




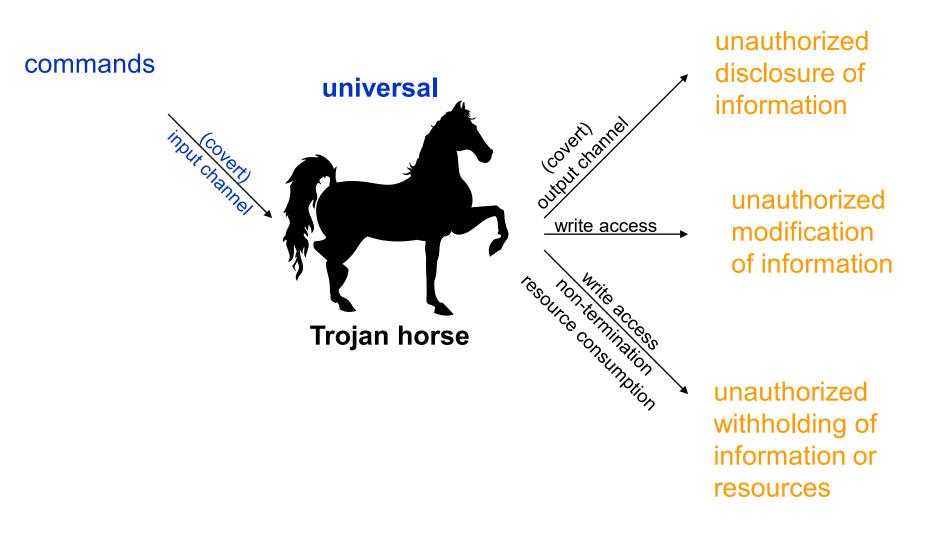




Transitive propagation of errors and attacks









Protection against whom ?

Laws and forces of nature

- components are growing old
- excess voltage (lightning, EMP)
- voltage loss
- flooding (storm tide, break of water pipe, heavy rain)
- change of temperature ...

Human beings

- outsider
- user of the system
- operator of the system
- -service and maintenance
- -producer of the system
- -designer of the system
- -producer of the tools to design and produce
- designer of the tools to design and produce
- -producer of the tools to design and produce the tools to design and produce
- -designer... includes

user.

operator,

fault tolerance

Trojan horse universal transitive

service and maintenance ... of the system used



Which protection measures against which attacker ?

protection concerning protection against	to achieve the intended	to prevent the unintended
designer and producer of the tools to design and produce	intermediate languages and intermediate results, which are analyzed independently	
designer of the system	see above + several independent designers	
producer of the system	independent analysis of the product	
service and maintenance	control as if a new product, see above	
operator of the system		restrict physical access, restrict and log logical access
user of the system	physical and logical restriction of access	
outsiders	protect the system physically and protect the data cryptographically from outsiders	



Which protection measures against which attacker ?

protection concerning protection against	to achieve the intended	to prevent the unintended
designer and producer of the tools to design and produce	intermediate languages a which are analyze	
designer of the system	see above + several ir	ndependent designers
producer of the system	independent analy	sis of the product
service and maintenance	control as if a new	product, see above
operator of the system		restrict physical access, restrict and log logical access
user of the system	physical and logical	restriction of access
outsiders	protect the system phy cryptographicall	

physical distribution and redundance

confidentiality, unobservability, anonymity, unlinkability:

avoid the ability to gather "unnecessary data"





It's not possible to protect against an omnipotent attacker.



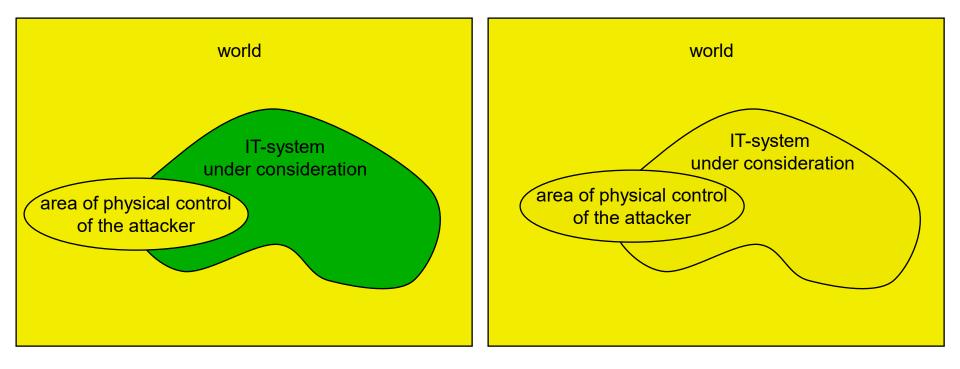
- area of physical control of the attacker
- behavior of the attacker

money

time

- passive / active
- observing / modifying (with regard to the agreed rules)
- stupid / intelligent
 - computing capacity:
 - not restricted: computationally unrestricted
 - restricted: computationally restricted





observing attacker

modifying attacker



acting according to the agreed rules



possibly breaking the agreed rules

Attacker (model) *A* is stronger than attacker (model) *B*, iff *A* is stronger than *B* in at least one respect and not weaker in any other respect.

Stronger means:

- set of roles of $A \supset$ set of roles of B,
- area of physical control of $A \supset$ area of physical control of B,
- behavior of the attacker
 - active is stronger than passive
 - modifying is stronger than observing
- intelligent is stronger than stupid
 - computing capacity: not restricted is stronger than restricted
- more money means stronger
- more time means stronger

Defines partial order of attacker (models).



confidentiality

- message content is confidential
- place sender / recipient anonymous

integrity

• time {

- detect forgery
- recipient can prove transmission
 sender can prove transmission
- ensure payment for service

availability

enable communication

authentication system(s) sign messages receipt during service by digital payment systems

diverse networks; fair sharing of resources

end-to-end encryption mechanisms to protect traffic data



- Each party has its particular protection goals.
- Each party can formulate its protection goals.
- Security conflicts are recognized and compromises negotiated.
- Each party can enforce its protection goals within the agreed compromise.

Security with minimal assumptions about others

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Security with minimal assumptions about others





- Each party has its particular goals.
- Each party can formulate its protection goals.
- Security conflicts are recognized and compromises negotiated.
- Each party can enforce its protection goals within the agreed compromise. As far as limitations of this cannot be avoided, they equally apply to all parties.

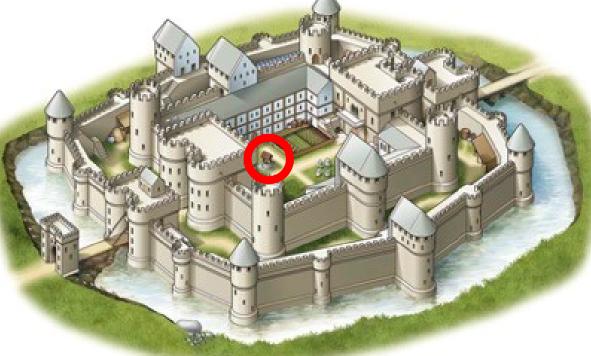
Security with minimal assumptions about others

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Multilateral Security vs. "Zero Trust"

[Rory Ward, Betsy Beyer: "BeyondCorp: A New Approach to Enterprise Security", 2014]

- marketing term "zero trust"
- fundamental idea:
 - trustworthy systems with minimal trust assumptions about all involved entities
- «Zero Trust Cybersecurity: 'Never Trust, Always Verify' » (NIST)
- current praxis:
 - perimeter security
 - firewall-like
 - "bad" outside
 - "good" inside





Each technical security measure needs a physical "anchoring" in a part of the system which the attacker has neither read access nor modifying access to.

Range from "computer centre X" to "smart card Y"

What can be expected at best ?

Availability of a locally concentrated part of the system cannot be provided against *realistic* attackers

\rightarrow physically distributed system

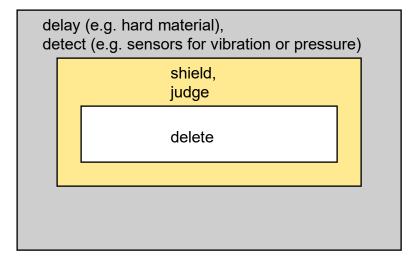
... hope the attacker cannot be at many places at the same time.

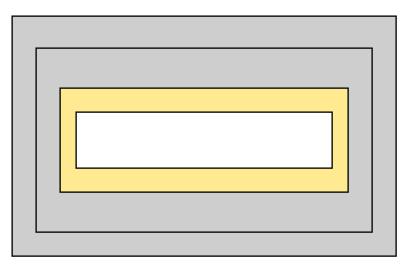
Distribution makes **confidentiality** and **integrity** more difficult. But physical measures concerning confidentiality and integrity are more efficient: Protection against *all realistic* attackers seems feasible. If so, physical distribution is quite ok.

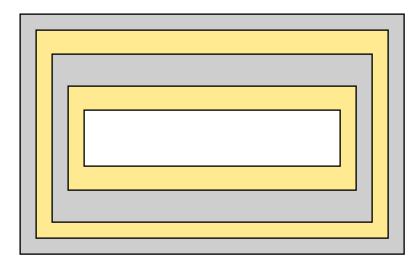


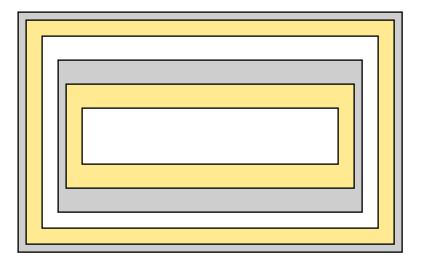


Shell-shaped arrangement of the five basic functions











Interference: detect judge

- Attack: delay delete data (etc.)
- Possibility: several layers, shielding
- Problem: validation ... credibility

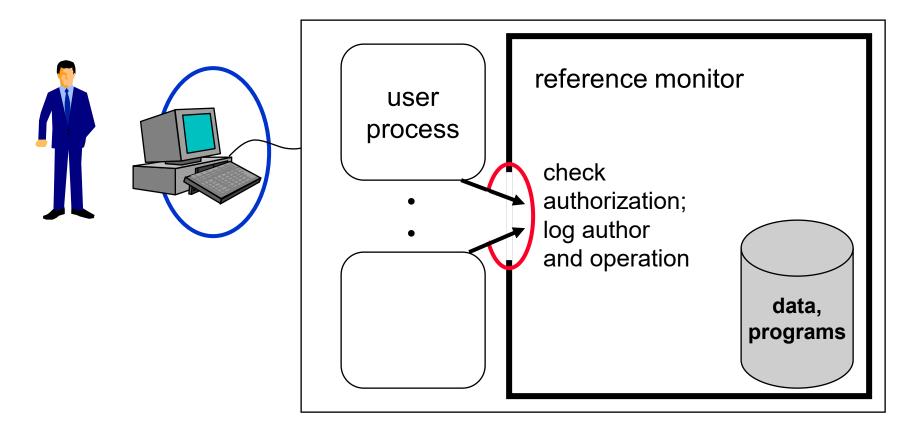
Negative example: smart cards

- no detection (battery missing etc.)
- shielding difficult (card is thin and flexible)
- no deletion of data intended, even when power supplied



Correspondence between organizational and IT structures

Admission control communicate with authorized partners only

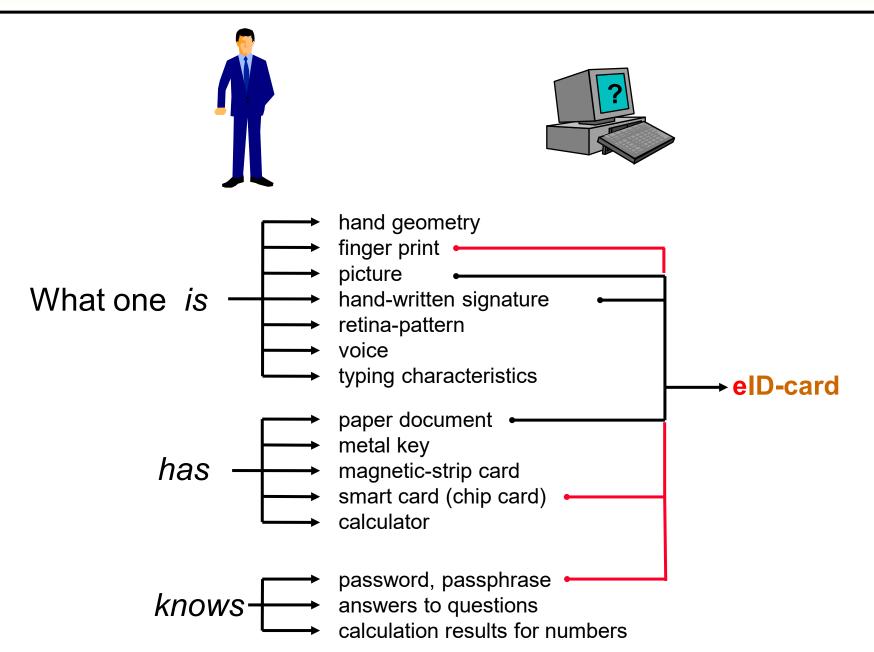


Access control

subject can only exercise operations on objects if authorized.



Identification of human beings by IT-systems



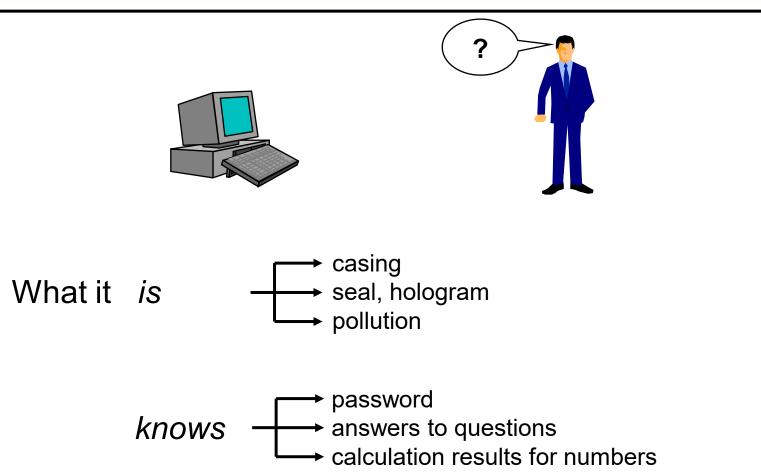
New German eID Card



PIN protects access to chip



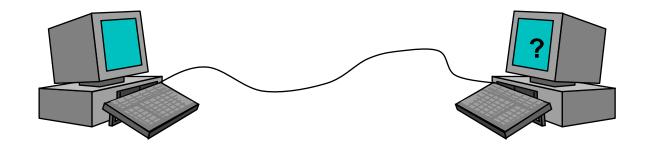
Identification of IT-systems by human beings

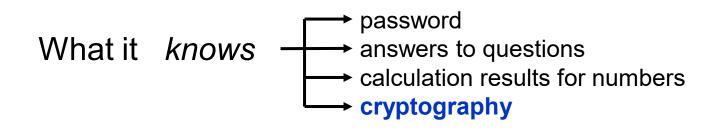


Where it *stands*

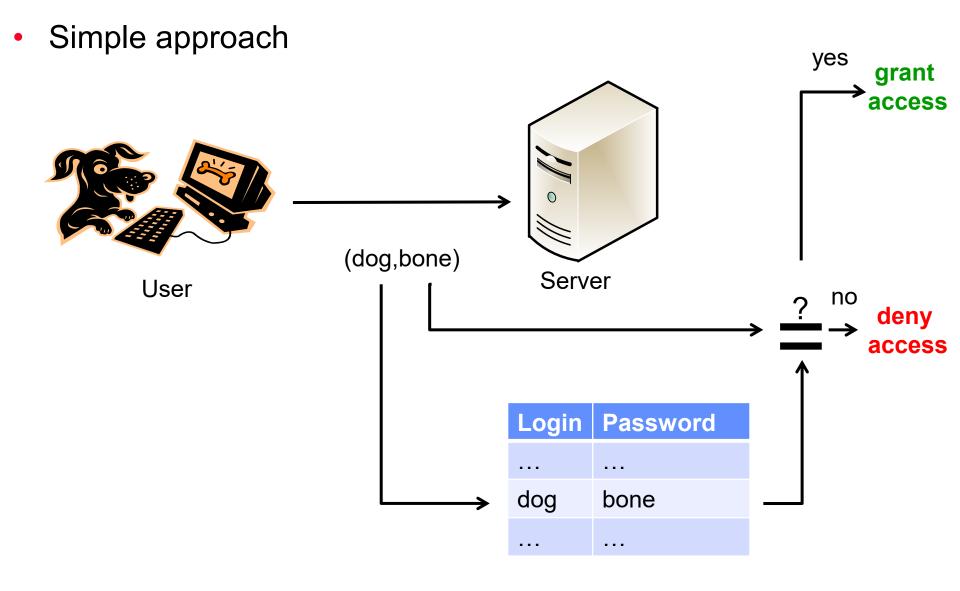


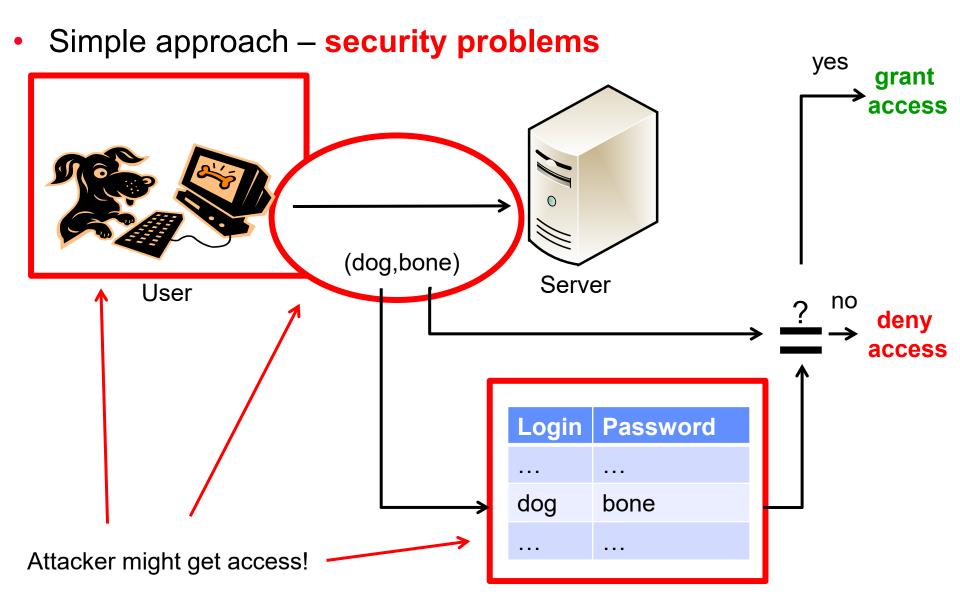
Identification of IT-systems by IT-systems





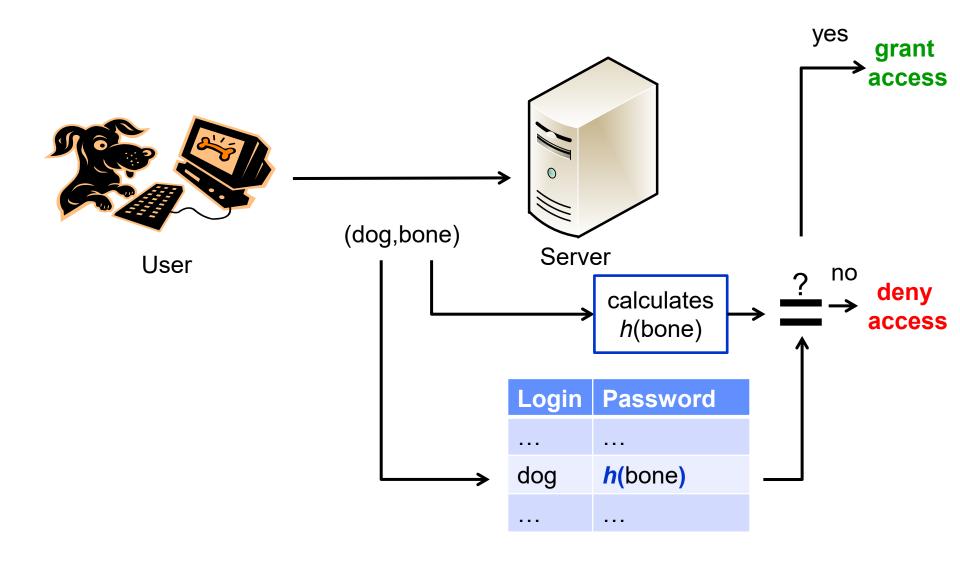
Wiring from where





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Enhanced approach using one way (hash) functions



One-way functions – cryptographic hash functions

- One-way function *f*:
 - calculating f(x)=y is easy
 - calculating $f^1(y)=x$ is hard
 - computation / storage
 - open question: Do one-way functions exist?
- Cryptographic hash function *h*
 - might have different properties depending on the use case
 - collision resistance:
 - it is hard to find x, y with h(y)=h(x) and $y\neq x$
 - note: *h* is usually not *collision free*, because $|h(x)| \ll |x|$
 - preimage resistance / one-way function / secrecy
 - given *h*(x) it is hard to find *x*
 - second-preimage resistance / weak collision resistance / binding
 - given x, h(x) it is hard to find y with h(y)=h(x) and $y\neq x$
 - Note:
 - *h* is not necessarily a "random extractor"
 - only one of "secrecy" and "binding" can be information theoretic secure

Examples for cryptographic hash functions

• MD5

- Message-Digest Algorithm
- developed by Ronald Rivest (April 1992)
- produces 128 bit hash values
- can process arbitrary long inputs
- today MD5 is broken!
- SHA-1
 - Secure Hash Standard
 - published 1993 as FIPS PUB 180 by US NIST
 - produces 160 bit hash values
 - today SHA-1 is insecure!
- SHA-2
 - set of hash functions, with hash values of 224, 256, 384, 512 bit
 - published 2001 as FIPS PUB 180-2 by NIST
 - SHA-2 hash functions are believed to be secure
- SHA-3
 - will be the result of the NIST Cryptographic Hash Algorithm Competition started November 2007
 - 3 selection rounds, 5 finalists
 - October 2012: Keccak is winner
 - FIPS 202: "SHA-3 Standard: Permutation-Based Hash and Extendable-Output Functions" (08/15)

MD5 Hash in the Wild

- United States Cyber Command (USCYBERCOM)
 - mission statement:= "USCYBERCOM plans, coordinates, integrates, synchronizes and conducts activities to: direct the operations and defense of specified Department of Defense information networks and; prepare to, and when directed, conduct full spectrum military cyberspace operations in order to enable actions in all domains, ensure US/Allied freedom of action in cyberspace and deny the same to our adversaries."



MD5 Hash in the Wild

mission statement:= "USCYBERCOM plans, coordinates, integrates, synchronizes and conducts activities to: direct the operations and defense of specified Department of Defense information networks and; prepare to, and when directed, conduct full spectrum military cyberspace operations in order to enable actions in all domains, ensure US/Allied freedom of action in cyberspace and deny the same to our adversaries."



Oh

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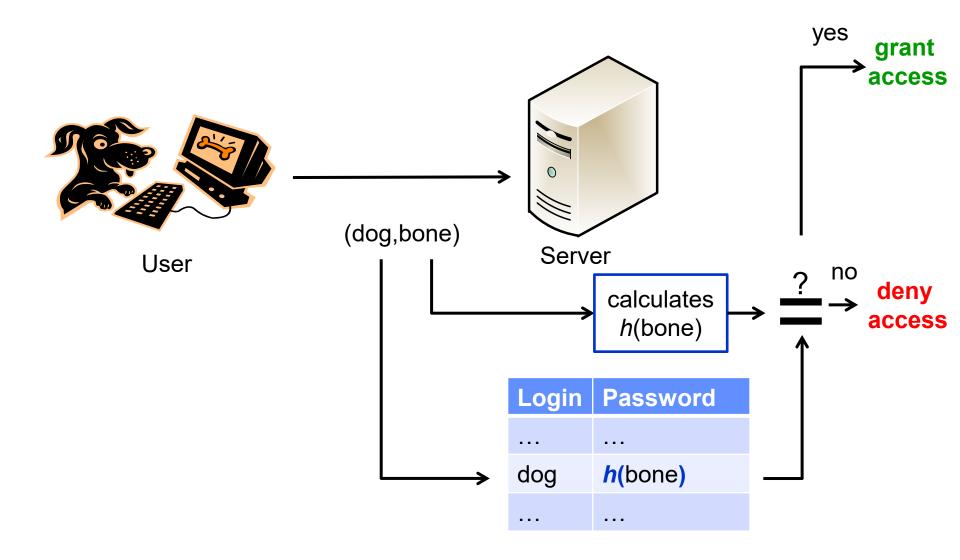
nD.

N

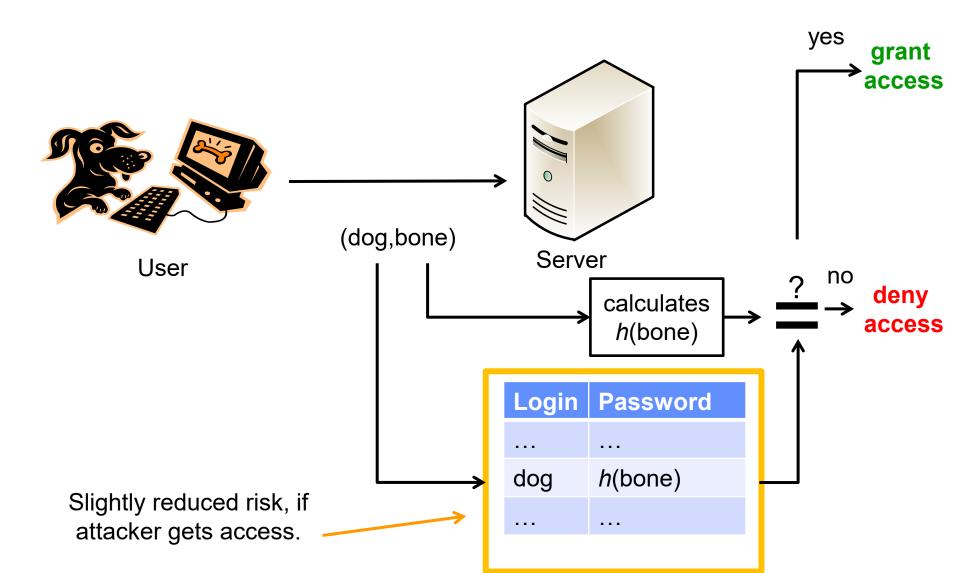
MD5(mission statement)= 9ec4c12949a4f31474f299058ce2b22a

(Remember: MD5 is broken \rightarrow find other interesting mission statements...)

Enhanced approach using one way (hash) functions



Enhanced approach using one way (hash) functions



Remaining problems of password based authentication based one way functions

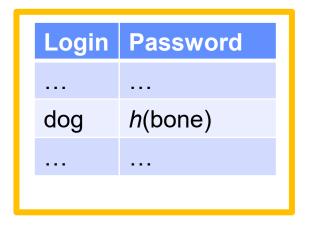
- Brute Force attack
 - function h() is public
 - value of h(x) is known to the attacker
 - \rightarrow try all possible values for x

Considerations:

- usually >> 1 Mio. *h*(x)/s on ordinary hardware
- assumption: password uses only small letters
- password length = 8

time needed:
$$\frac{26^8}{1\,000\,000\cdot 60\cdot 60} \approx 58h$$

- first countermeasures:
 - limit false attempts
- first password rules:
 - use a large alphabet (small and capitalised letters, numbers, specials)
 - use a long password



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Remaining problems of password based authentication ⁸¹ based one way functions

- first password rules:
 - use a large alphabet
 - (small, capitalised letters, numbers, specials)
 - time needed:
- $\frac{(26+26+10+30)^8}{1\,000\,000\cdot60\cdot60\cdot24\cdot365.25} \approx 162a$
- use a long password
- remaining possible attacks:
- increase in computation power
 - distributed approach
 - GPU
 - Moore's law
 - pre-computation:
 - attacker creates lockup table
 - search time (example above): $ld((26 + 26 + 10 + 30)^8) < 53$ comparisons

Login	Password
dog	<i>h</i> (bone)

Remaining problems of password based authentication based one way functions

- remaining possible attack:
 - pre-computation
- countermeasure:
 - salt!
 - $h(x) \rightarrow h(salt,x)$
 - salt:
 - long (e.g. 128 bit) random value
 - some part is unique for the system (i.e. 104 bit)
 - some part is randomly chosen by the system for each entry in the password table (i.e. 24 bit)
 - NOT stored at the system
 - verification: iterate over all possible salt values
 - ➔ pre-computation has to be done for each possible salt

Login	Password
dog	<i>h</i> (bone)

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Remaining problems of password based authentication

- based one way functions
- remaining possible attack:
 - dictionary attack
 - problem: people do not chose passwords randomly
 - often names, words or predictable numbers are used

Login	Password
dog	<i>h</i> (salt,bone)

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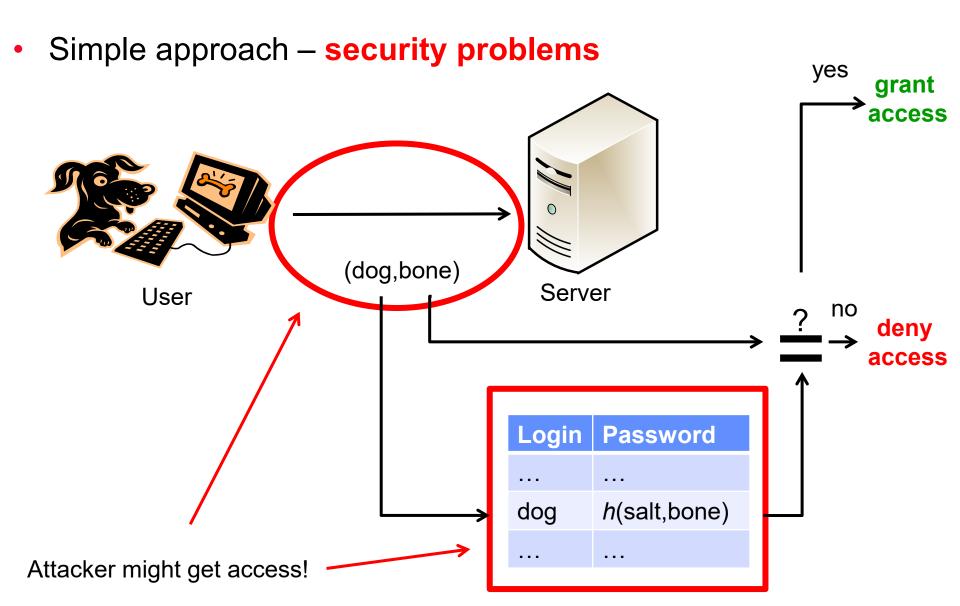
- <u>http://www.whatsmypass.com/the-top-500-worst-passwords-of-all-time</u>
- attacker uses dictionaries for brute force attack
- prominent program: John the Ripper
 - supports dictionary attacks and password patterns
- possible solutions:
 - enforce password rules
 - consider usability
 - pre-check passwords (e.g. using John)
 - train people to "generate" good passwords
 - Example: sentence \rightarrow password
 - "This is the password I use for Google mail" \rightarrow "Titplu4Gm"

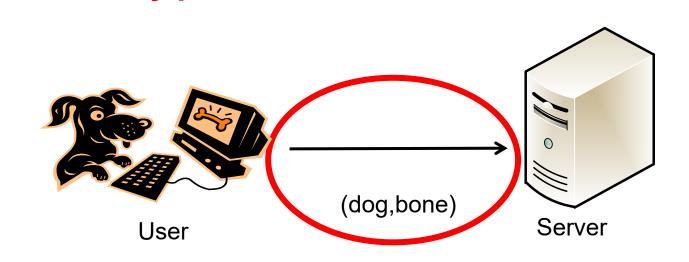
The Server as Attacker



- ... a new Web 2.0 service
- ... for people which like city journeys
- ... find cool cities and places like shops, restaurants, hotels etc.
- ... information from globe-trotters for globe-trotters
- ... they can share their knowledge after secure login
- So that's wrong?
- It collects (username,password) and tries to login into other popular services like Gmail, Twitter, eBay, Amazon etc.

password rule: never "reuse" passwords!

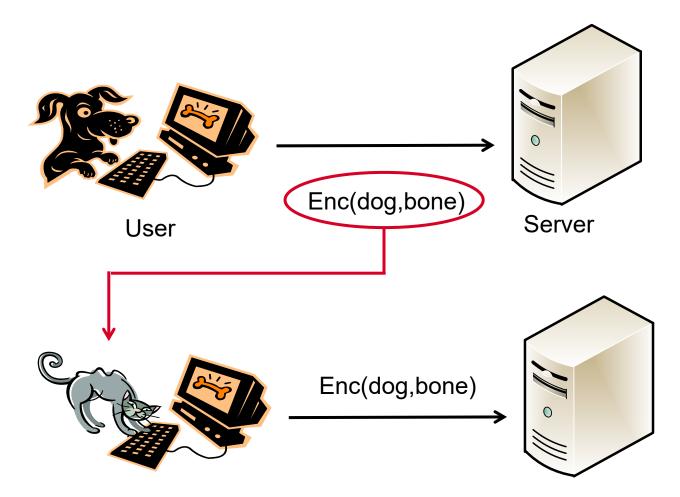




- possible solution:
 - encrypt communication
- remaining problems:

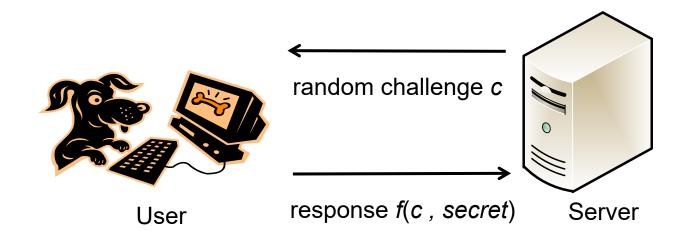
security problems

- not always possible
- replay attack

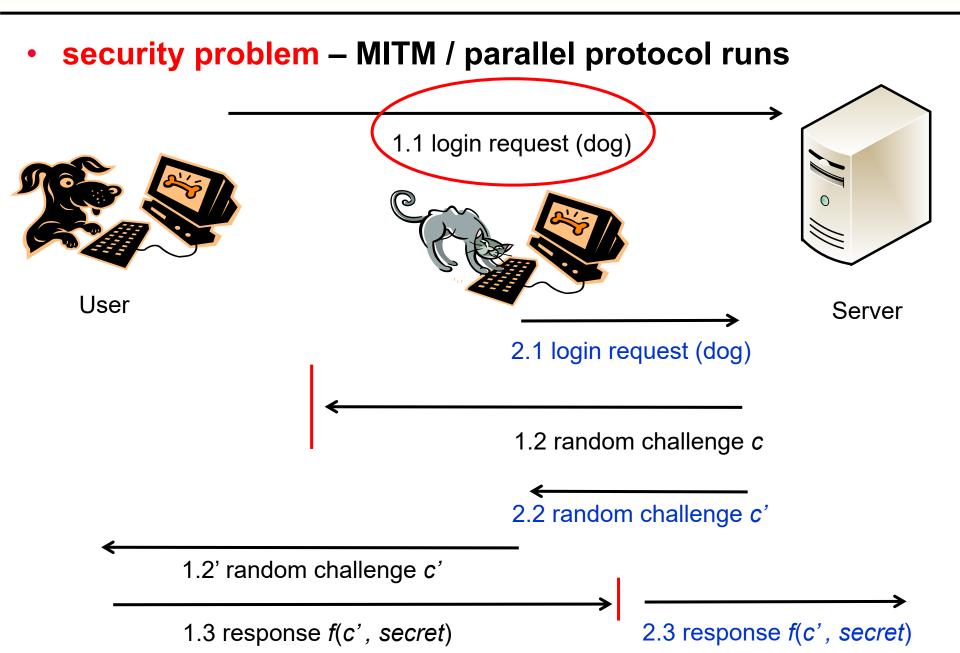


• security problem – replay attack

- security problem replay attack
- possible solution: challenge-response protocol

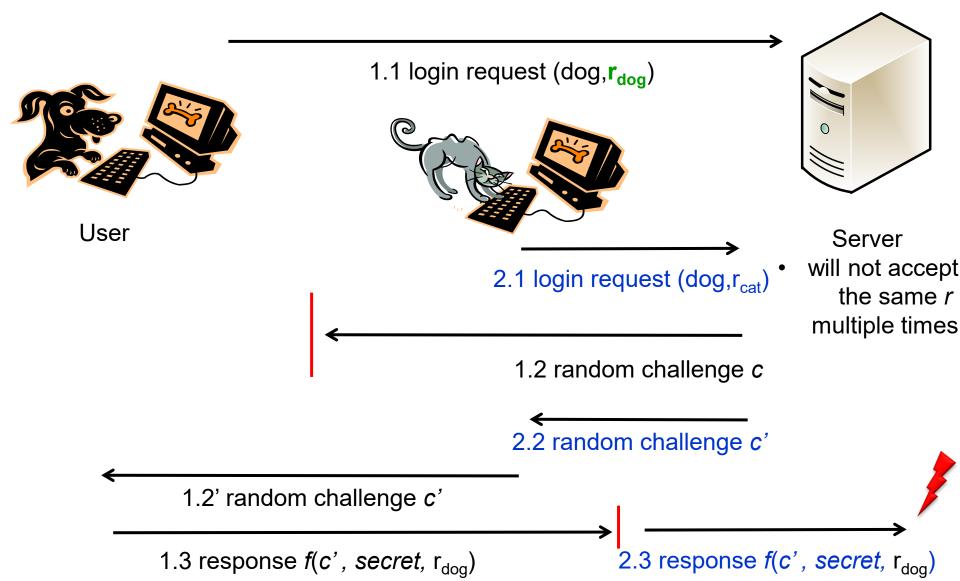


- tries to ensure freshness
- remaining problems:
 - Man-in-the-middle attacks
 - parallel protocol runs



- security problem MITM / parallel protocol runs
- possible solutions:
 - disallow parallel login protocol runs for the same user
 - make protocol runs distinguishable

possible solution: distinguishable protocol runs

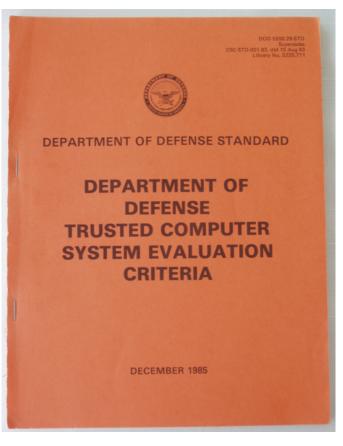


- security problem MITM / parallel protocol runs
- possible solutions:
 - disallow parallel login protocol runs for the same user
 - make protocol runs distinguishable
- remaining security problems:

(ok I will stop here – if you are interested in many more problems / solutions I recommend: Colin Boyd, Anish Mathuria: "Protocols for Authentication and Key Establishment", Springer, 2003.)

• (non protocol related) security problems:

- phising, i.e. faked UI for entering secret information
- today: mostly Internet based attacks
- but: local attacks possible as well
 - faked login / lock screen
 - solution: "trusted path" / Secure Attention Key 3.2.2.1.1 The TCB [Trusted Computing Base] shall support a trusted communication path between itself and user for initial login and authentication. Communications via this path shall be initiated exclusively by a user.
 [Department of Defense: "Trusted Computer System Evaluation Criteria", CSC-STD-001-83, 15. August 1983 – called "Orange Book"]
 - well known implementations:
 - Windows: Ctrl+Alt+Del
 - Linux: Ctrl+Alt+Pause
 - could be freely chosen in principle



http://en.wikipedia.org/wiki/File:Orange-book-small.PNG

- One Time Password
 - only used to authenticate a single transation
- Advantage
 - abuse of OTP becomes harder for the attacker
- Implementations
 - list of OTPs
 - known from online banking: TAN, iTAN
 - on the fly generated and transmitted over a second channel
 - mTAN
 - time-synchronized (hardware) tokens:
 - token knows a secret s
 - OTP= *f*(s,time)
 - hash chain based

OTP Implementations

- hash chain based
 - Leslie Lamport: "Password Authentication with Insecure Communication"
 - users generates hash chain:

 $-h^{n}(\dots h^{3}(h^{2}(h^{1}(\text{password}))))$

- users sends hⁿ() as his "password" during register procedure
- next login user sends hⁿ⁻¹()
- server verifies: h(*h*ⁿ⁻¹()) = *h*ⁿ()
- server now stores: hⁿ⁻¹()

- Physiological or behavioural characteristics (of a human being) are measured and compared with reference values to
 - verify, that a given subject is the one it claimed to be
 - claimed "identity" is known to the system by other means
 - identify, a subject within a given set of (known) subjects
 - "identity" should be derived from biometrics
 - usually more difficult compared to verification

Biometrics: Physiological / Behavioural Characteristics



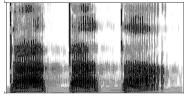
Iris / Retina



Fingerprint



Hand geometry



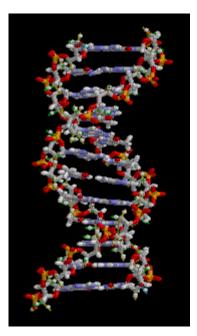
Voice spectrogram



(3D) Face geometry



Handwriting: appearance, dynamics of writing



DNA



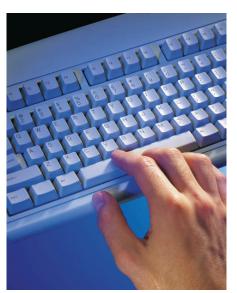
Gait

[Pictures are mostly from Wikipedia]

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Thermography: facial thermograms



Key strokes: dynamics of writing (speed, pressure etc.)

Biometric characteristics: Requirements

- universal: everyone has it
- unique
- stable over time
- measurable
- acceptable
- analysable
- resistant against cloning / faking

• Pros:

- Cannot be divulged or lost/forgotten
- can be utilised "on the fly"
- Hard to copy

Cons:

- Cannot be renewed
- Person related data requires special protection (privacy)
- Invasion (of privacy)
- Error rate



- Pros:
 - Cannot be divulged or lost/forgotten
 - but could be stolen

Safety Risks of Biometrics

Demolition Man (1993): Simon Phoenix (Wesley Snipes) escaping from the jail...

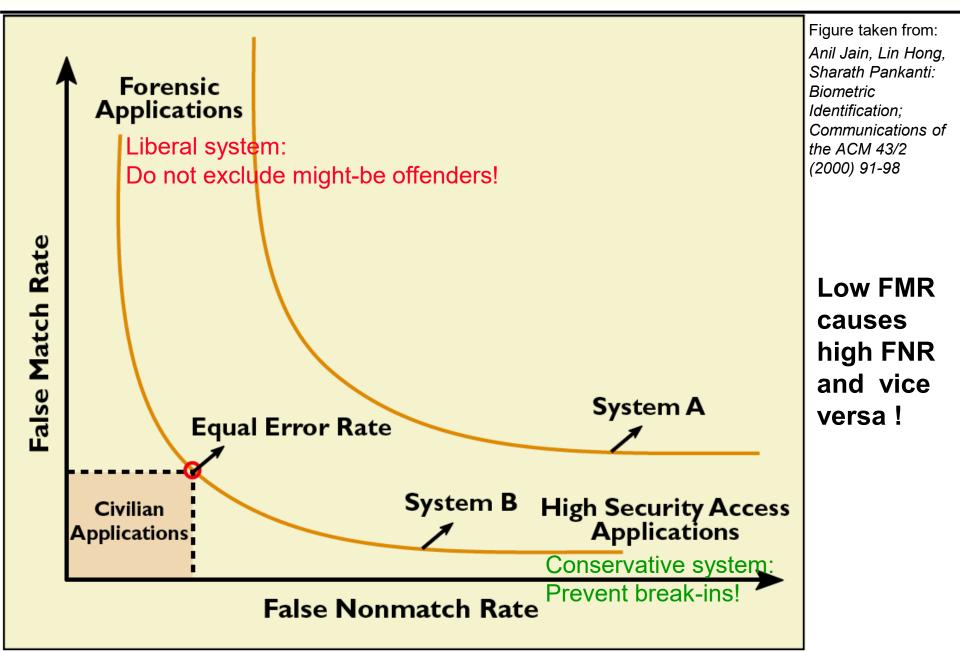
- Pros:
 - Cannot be divulged or lost/forgotten
 - but could be stolen:
 - http://news.bbc.co.uk/2/hi/asia-pacific/4396831.stm
 - could become "unusable" due to
 - ageing
 - incidents
 - disease
 - can be utilised "on the fly"
 - privacy problems (unnoticeable measurement of Biometrics)
 - Hard to copy
 - depends on the Biometric system used
 - many systems are easy to cheat
 - <u>ftp://ftp.ccc.de/pub/documentation/Fingerabdruck_Hack/fingerabdruck.</u> mpg

Demonstration of Fingerprint Cloning by CCC

- Pros:
 - Cannot be divulged or lost/forgotten
 - but could be stolen:
 - http://news.bbc.co.uk/2/hi/asia-pacific/4396831.stm
 - could become "unusable" due to
 - ageing
 - incidents
 - disease
 - can be utilised "on the fly"
 - privacy problems (unnoticeable measurement of Biometrics)
 - Hard to copy
 - depends on the Biometric system used
 - many systems are easy to cheat
 - <u>ftp://ftp.ccc.de/pub/documentation/Fingerabdruck_Hack/fingerabdruck.</u> <u>mpg</u>
 - cloning of e.g. fingerprints might be in the interest of law enforcement
 - access to biometrically secured devices

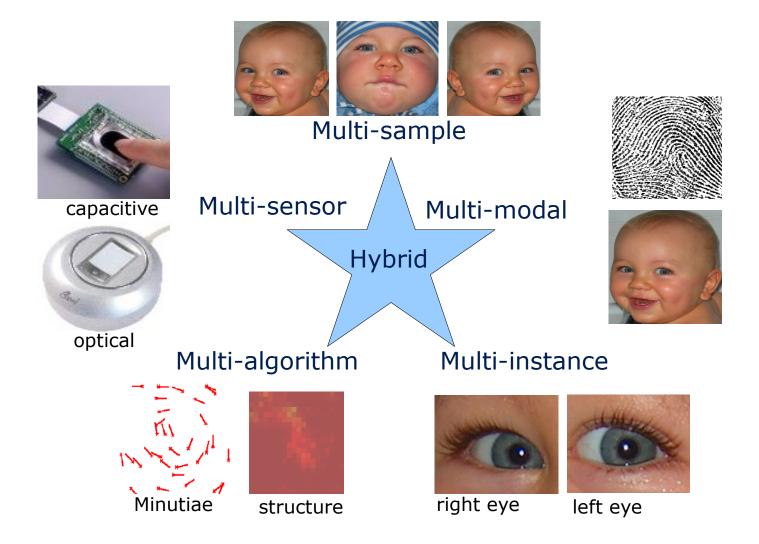
- False Accept Rate (FAR) / False Match Rate (FMR):
 Security problem!
- False Reject Rate (FRR) / False Nonmatch Rate (FNR):
 - Usability / acceptance problem
- Receiver Operating Characteristic (ROC):
 - curve of FAR against FRR
- Equal Error Rate (EER):
 - rate for FAR=FRR
 - can be seen from the ROC curve





- False Accept Rate (FAR):
 - Security problem!
- False Reject Rate (FRR):
 - Usability / acceptance problem
- Receiver Operating Characteristic (ROC):
 - curve of FAR against FRR
- Equal Error Rate (EER):
 - error rate for FAR=FRR
 - can be seen from the ROC curve
- Failure To Enroll Rate (FTE):
 - Usability / acceptance problem
- Failure To Capture Rate (FTC):
 - Usability / acceptance problem

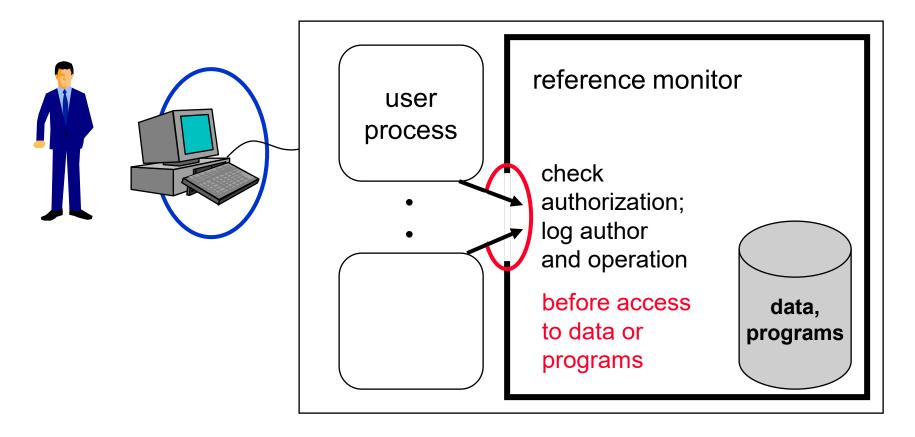
Enhanced Security: Multi-biometric Systems



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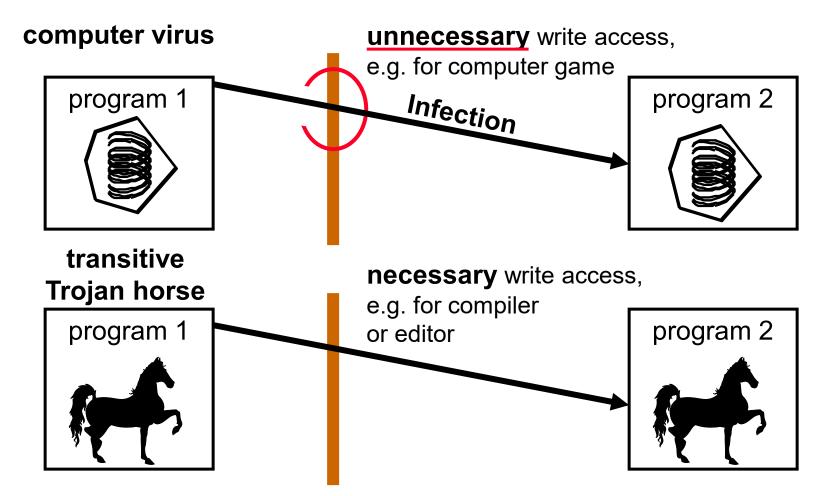
Admission control communicate with authorized partners only



Access control

subject can only exercise operations on objects if authorized.

Computer virus vs. transitive Trojan horse



Access control

Limit spread of attack by as little privileges as possible: Don't grant unnecessary access rights!

No computer viruses, only transitive Trojan horses!



Other measures fail:

1. Undecidable if program is a computer virus proof (indirect) assumption: decide (•)

program counter_example
if decide (counter_example)

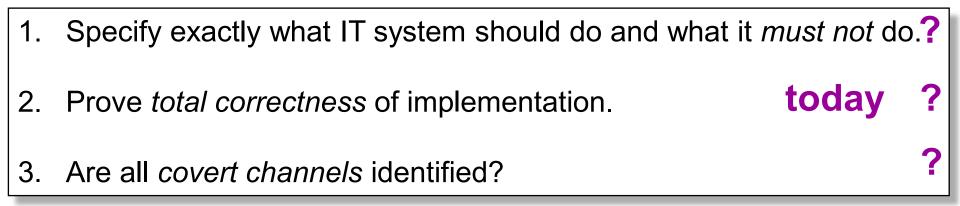
then no_virus_functionality
else virus_functionality

2. Undecidable if program is Trojan horse

Better be too careful!

- 3. Even known computer viruses are not efficiently identifiable self-modification virus seamer
- 4. Same for: Trojan horses
- 5. Damage concerning data is not ascertainable afterwards function inflicting damage could modify itself







Design and realize IT system as *distributed* system, such that a limited number of attacking computers cannot inflict significant damage.



Aspects of distribution

physical distribution distributed control and implementation structure

distributed system:

no entity has a global view on the system



Trustworthy terminals

Trustworthy only to user to others as well

Ability to communicate

Availability by redundancy and diversity

Cryptography

Confidentiality byencryptionIntegrity bymessage authentication codes (MACs) or digital signatures



Infrastructure with the least possible complexity of design

Connection to completely diverse networks

- different frequency bands in radio networks
- redundant wiring and diverse routing in fixed networks

Avoid bottlenecks of diversity

- e.g. radio network needs same local exchange as fixed network,
- for all subscriber links, there is only one transmission point to the long distance network



Achievable protection goals: confidentiality, called concealment integrity (= no *undetected* unauthorized modification of information), called authentication

Unachievable by cryptography: availability – at least not against strong attackers



Kerckhoff's Priniple
(1883 by Auguste Kerckhoffs: "La cryptographie militaire")

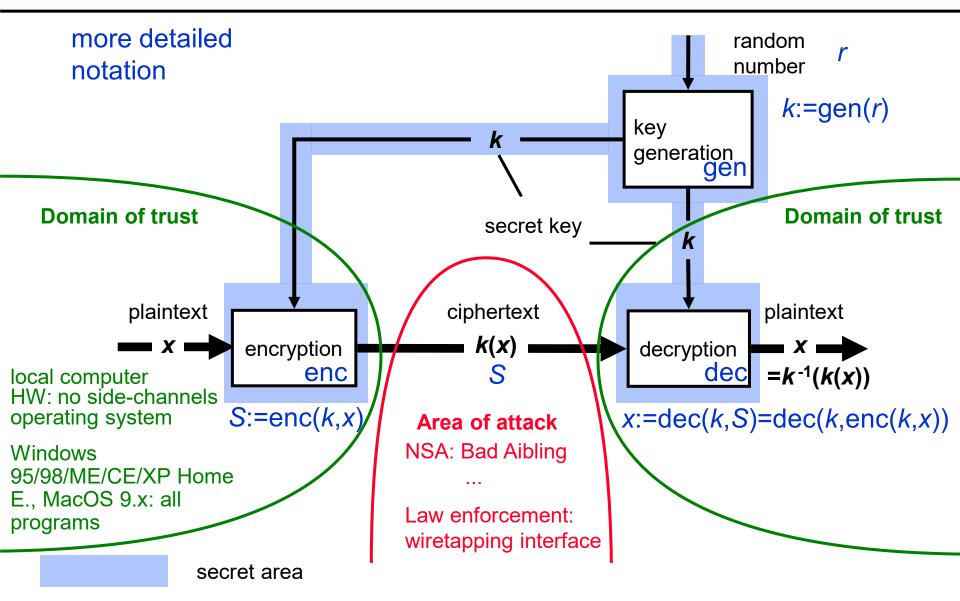
"The cipher method **must not** be required to be secret, and it must be able to fall into the hands of the enemy without inconvenience"

% Therefore:

- ➡ use only publicly known and well analysed algorithms and protocols
- ➣ do not trust "super secure" but secret algorithms
- ➢ do not design your own algorithms (at least as longs as you are not a cryptographic expert)
- ➢ Remember: Combining secure building blocks does not necessarily lead to a secure overall system!

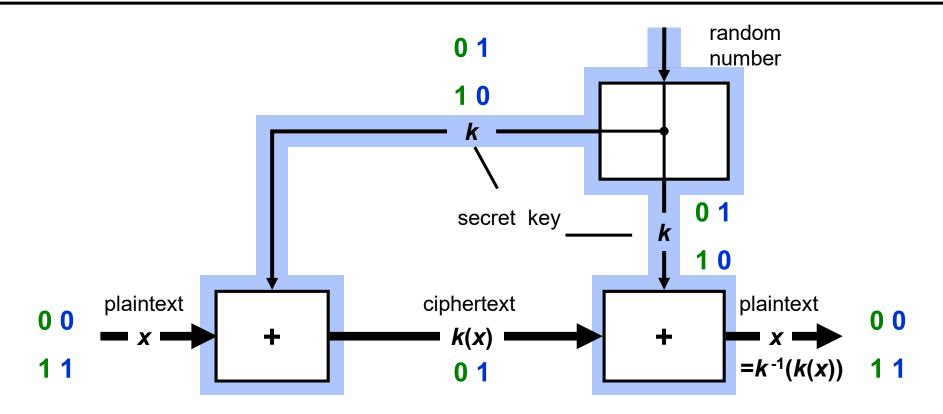


Symmetric encryption system



Opaque box with lock; 2 identical keys

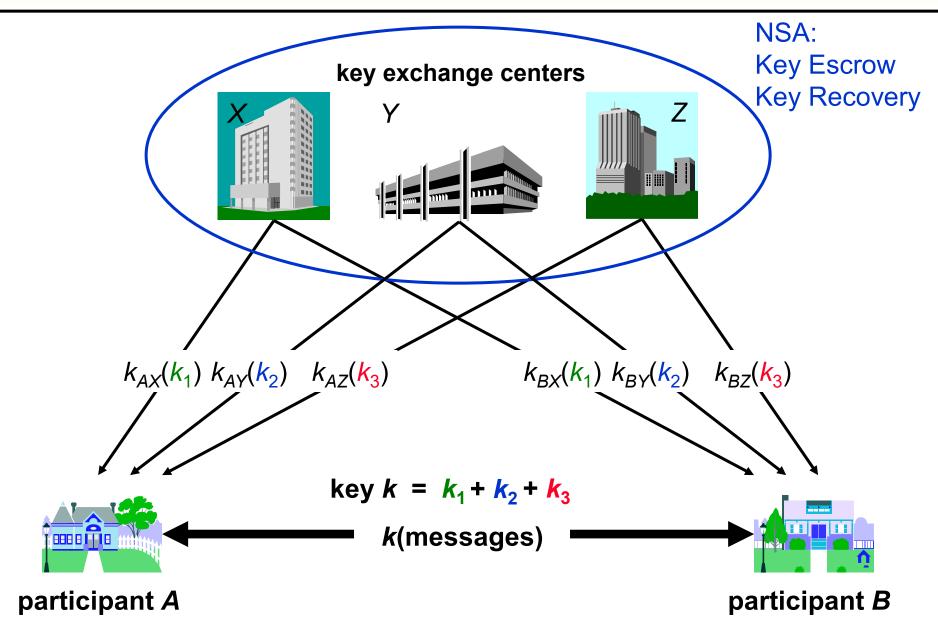
Example: Vernam cipher (=one-time pad)



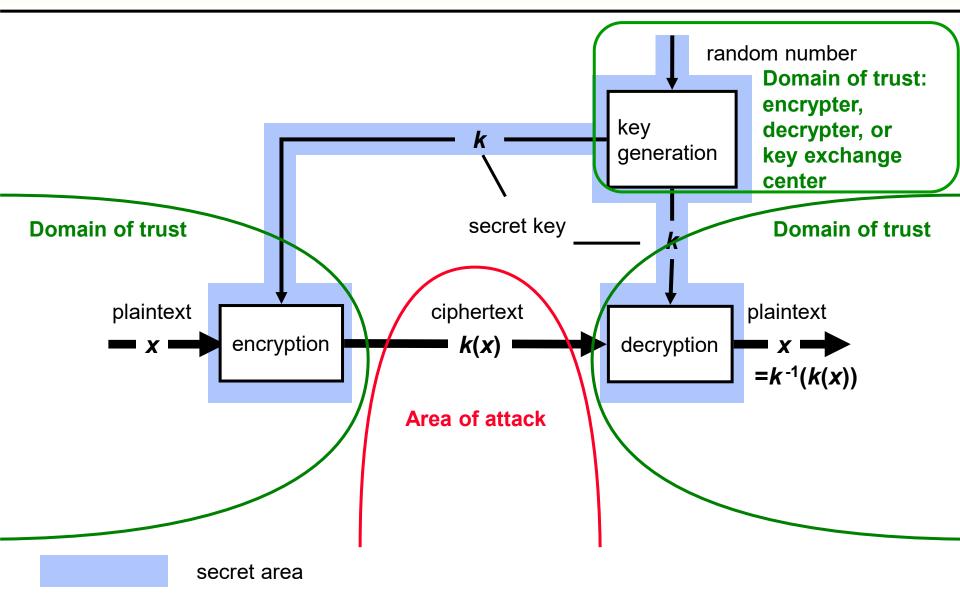
secret area

Opaque box with lock; 2 identical keys

Key exchange using symmetric encryption systems

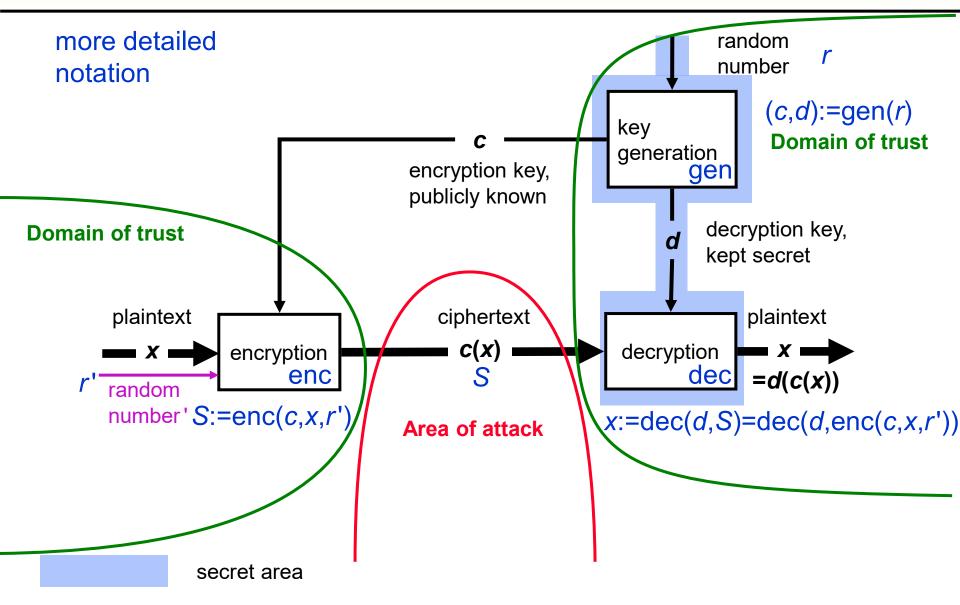


Sym. encryption system: Domain of trust key generation



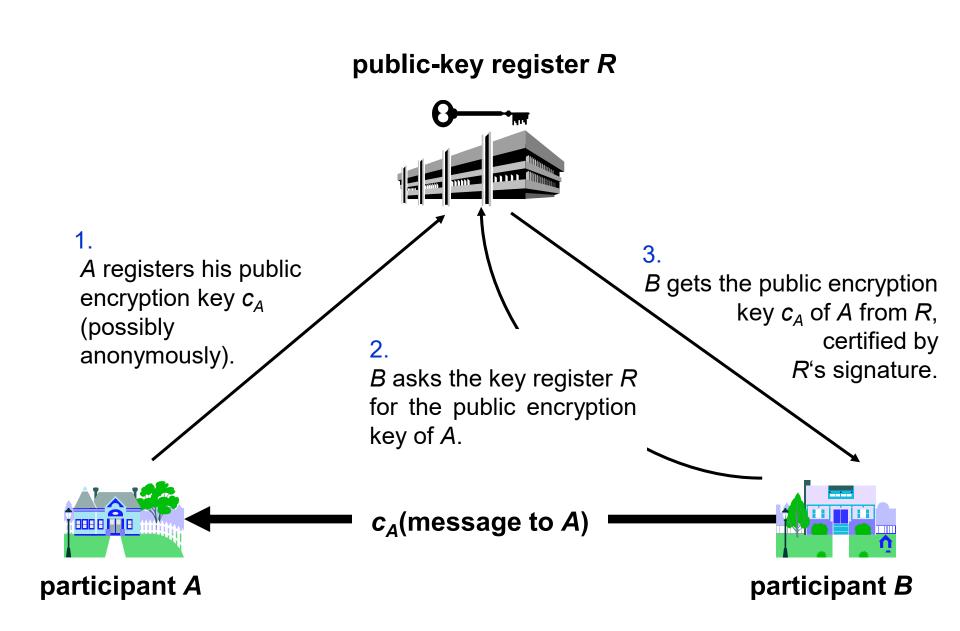


Asymmetric encryption system



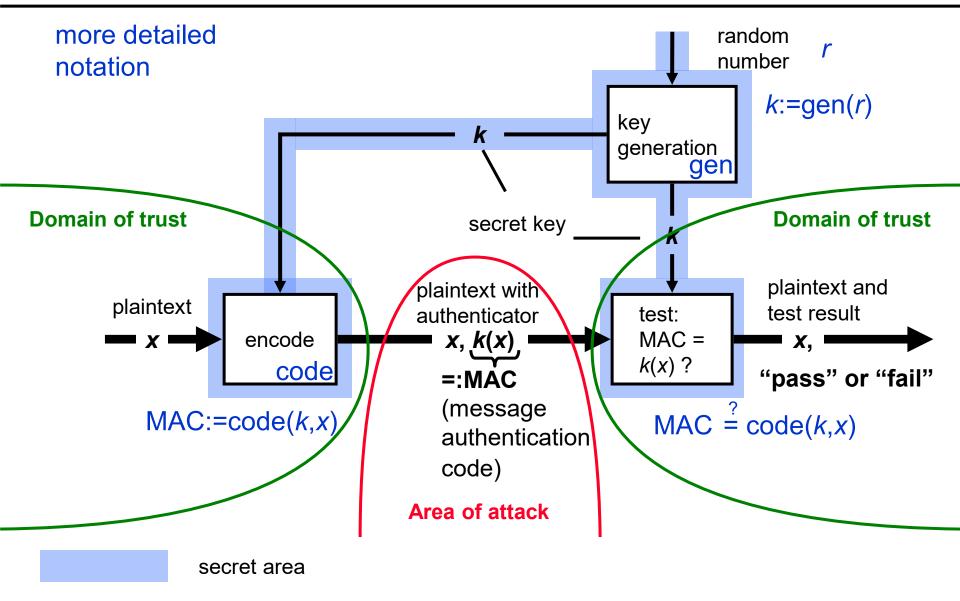
Opaque box with spring lock; 1 key

Key distribution using asymmetric encryption systems





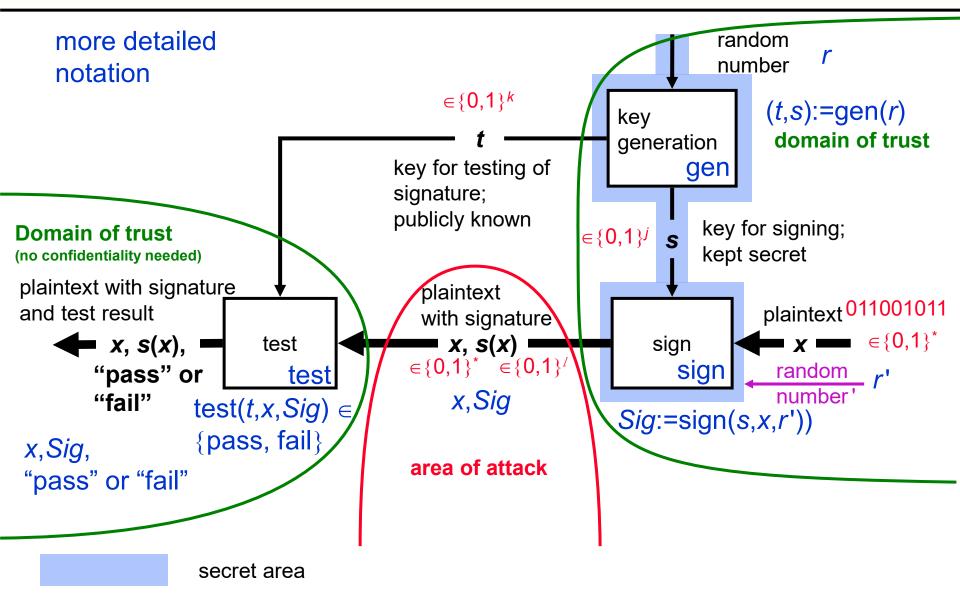
Symmetric authentication system



Show-case with lock; 2 identical keys

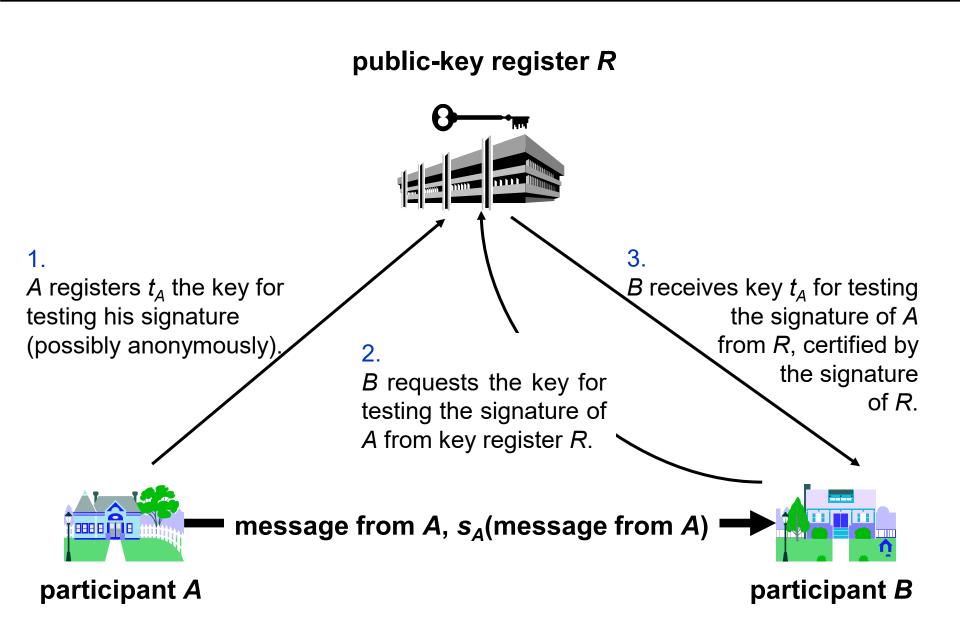


Digital signature system

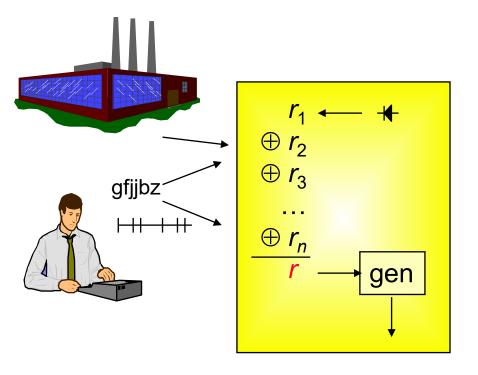


Show-case with lock; 1 key









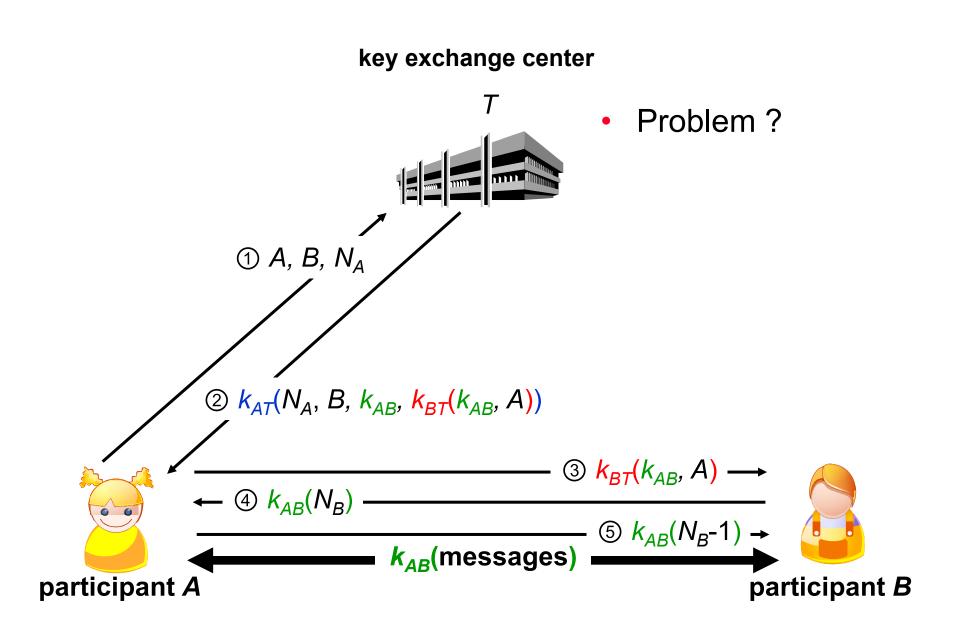
generation of a random number *r* for the key generation:

XOR of

- r_1 , created in device,
- r_2 , delivered by producer,
- r_3 , delivered by user,
- *r_n*, calculated from keystroke intervals.

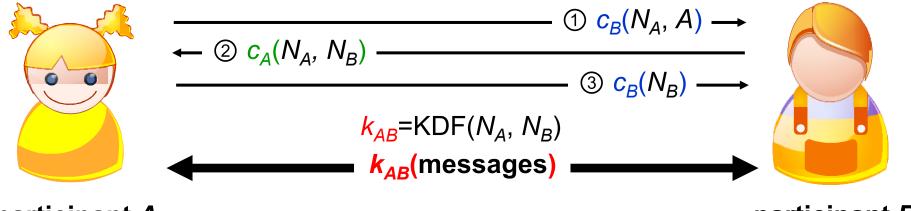
- from 1978
- goals:
 - key freshness:
 - key is "fresh", i.e. a newly generated one
 - key authentication:
 - key is only known to Alice and Bob (and maybe some trusted third party)
- preconditions:
 - a trusted third party T
 - shared term secret keys between Alice (resp. Bob) and the trusted third party:
 - *k*_{AT}, *k*_{BT}

Needham-Schroeder-Protocol using Symmetric encryption



- from 1978
- goals:
 - key freshness:
 - key is "fresh", i.e. a newly generated one
 - key authentication:
 - key is only known to Alice and Bob
- preconditions:
 - public encryption keys of Alice c_A and Bob c_B known to each other

Needham-Schroeder-Protocol using Asymmetric encryption



participant A

participant **B**

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• Problem ?



Whom are keys assigned to?

- 1. individual participants asymmetric systems
- 2. pair relations symmetric systems
- 3. groups

How many keys have to be exchanged?

n participantsasymmetric systemsn per systemsymmetric systems $n \cdot (n-1)$

When are keys generated and exchanged?

Security of key exchange limits security available by cryptography:

execute several initial key exchanges



a) key (total break)

b) procedure equivalent to key (universal break)

c) individual messages,

e.g. especially for authentication systems

- c1) one selected message (selective break)
- c2) any message (existential break)



severity

- a) passive
 - a1) ciphertext-only attack
 - a2) known-plaintext attack
- b) active

(according to encryption system; asym.: either b1 or b2;

- sym.: b1 or b2)
- b1) signature system: plaintext → ciphertext (signature) (chosen-plaintext attack)
- b2) encryption system: ciphertext → plaintext (chosen-ciphertext attack)
- adaptivity

not adaptive adaptive

criterion: action

permission

passive attacker \neq observing attackeractive attacker \neq modifying attacker



Basic facts about "cryptographically strong" (1)

If no security against computationally unrestricted attacker:

- 1) using of keys of constant length \mathcal{L} :
 - attacker algorithm can always try out all $2^{\mathcal{L}}$ keys (breaks asym. encryption systems and sym. systems in known-plaintext attack).
 - requires an exponential number of operations (too much effort for l > 100).
 - \rightarrow the best that the designer of encryption systems can hope for.
- 2) complexity theory:
 - mainly delivers asymptotic results
 - mainly deals with "worst-case"-complexity
 - \rightarrow useless for security; same for "average-case"-complexity.

goal: problem is supposed to be difficult almost everywhere, i.e. except for an infinitesimal fraction of cases.

- security parameter ${\cal L}$ (more general than key length; practically useful)

fašt

- if $\mathcal{U} \to \infty$, then probability of breaking $\to 0$.
- hope: slow



3) 2 classes of complexity:

en-/decryption: easy = polynomial in \mathcal{L} breaking: hard = not polynomial in $\mathcal{L} \approx$ exponential in \mathcal{L} Why?

- a) harder than exponential is impossible, see 1).
- b) self-contained: substituting polynomials in polynomials gives polynomials.
- c) reasonable models of calculation (Turing-, RAM-machine) are polynomially equivalent.

For practice polynomial of high degree would suffice for runtime of attacker algorithm on RAM-machine.

- 4) Why assumptions on computational restrictions, e.g., factoring is difficult? Complexity theory cannot prove any useful lower limits so far. Compact, long studied assumptions!
- 5) What if assumption turns out to be wrong?
 - a) Make other assumptions.

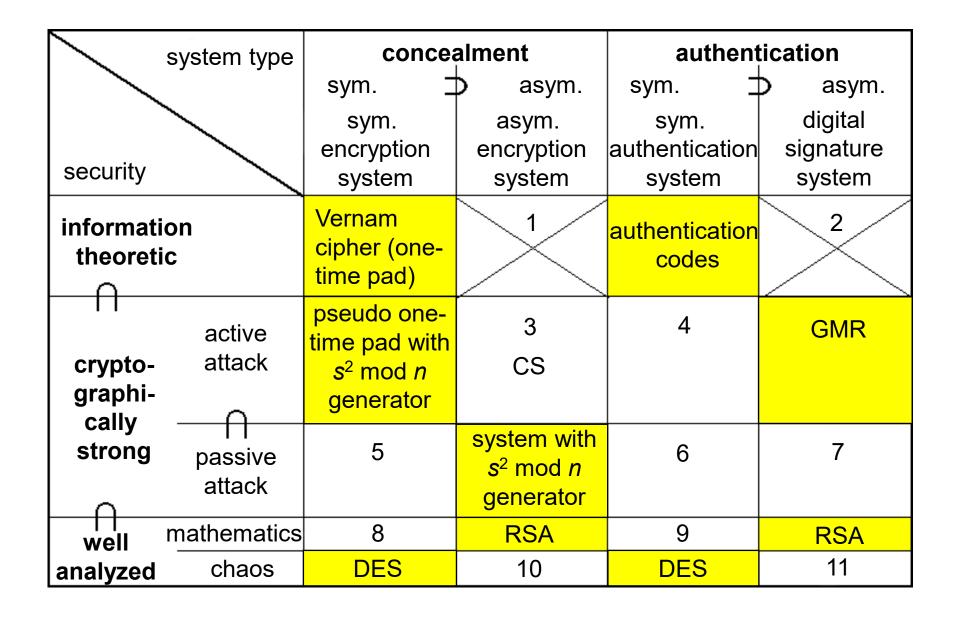
b) More precise analysis, e.g., fix model of calculation exactly and then examine if polynomial is of high enough degree.

6) Goal of proof: If attacker algorithm can break encryption system, then it can also solve the problem which was assumed to be difficult.









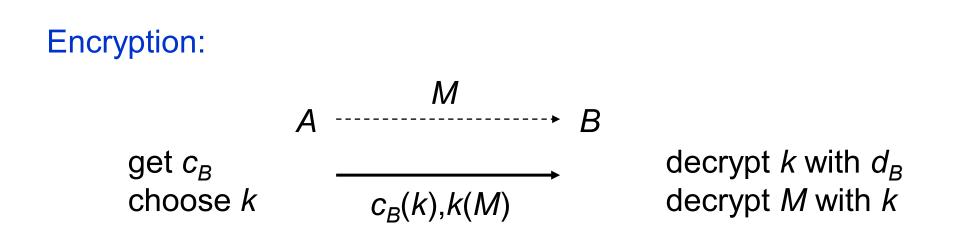


Combine:

- from asymmetric systems: easy key distribution
- from symmetric systems: efficiency (factor 100 ... 10000, SW and HW)

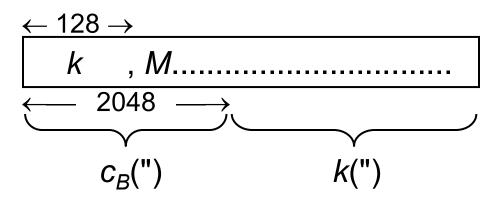
How?

use asymmetric system to distribute key for symmetric system





Even more efficient: part of *M* in first block

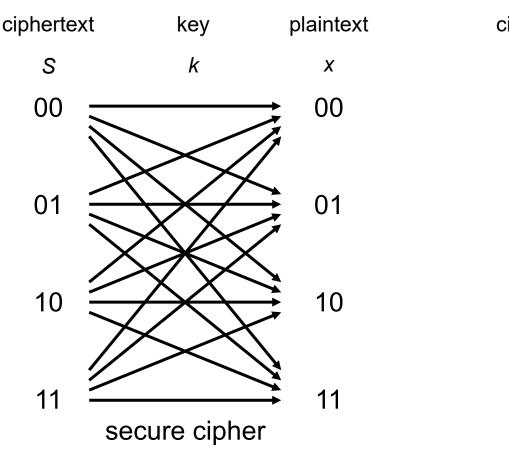


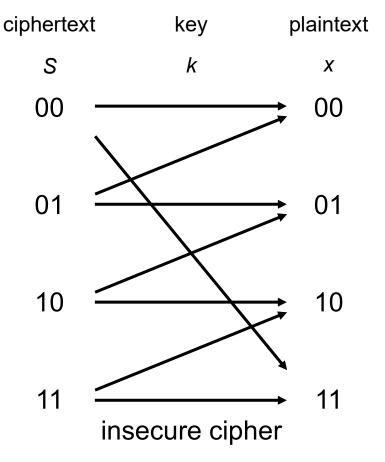
If *B* is supposed also to use *k*: append $s_A(B,k)$

Authentication: k authorized and kept secret

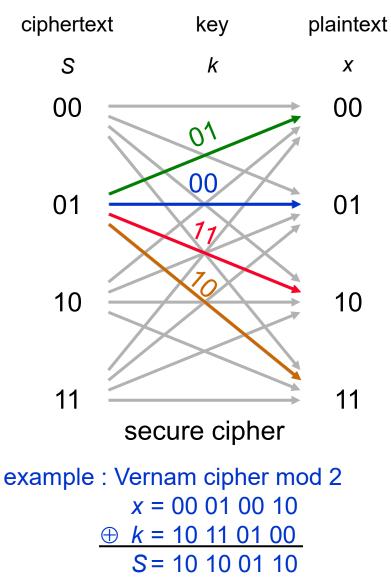
get c_B choose k $M,k(M),c_B(B,k,s_A(B,k))$ $decrypt c_B(B,k,s_A(B,k))$ MAC test B,k with t_A test M with k

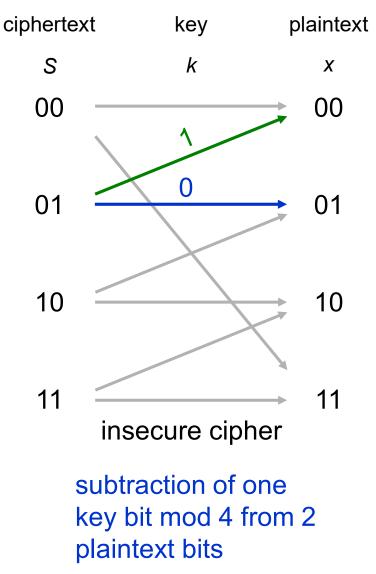
"Any ciphertext S may equally well be any plaintext x"



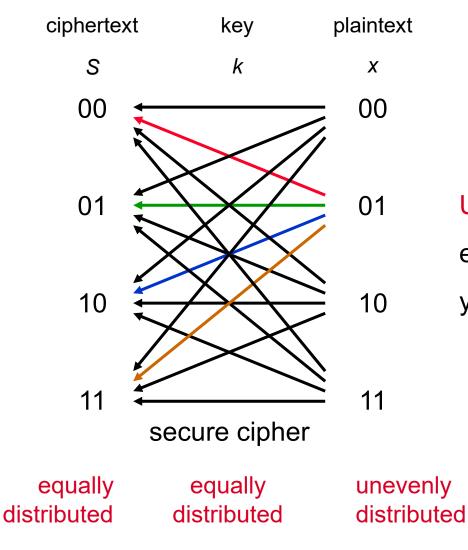


"Any ciphertext S may equally well be any plaintext x"





Different probability distributions – how do they fit?



Unevenly distributed plaintexts enciphered with equally distributed keys yield equally distributed ciphertexts.

Different probability distributions – how do they fit?

ciphertex	xt key	plaintext	
S	k	X	
00		00	
01		01	
10		10	
11	secure cipher	11	
equally distributed	equally distribu- ted, but <i>not</i> independently of the ciphertexts	unevenly distributed	

Equally distributed ciphertexts deciphered with equally distributed keys can yield unevenly distributed plaintexts, iff ciphertexts and keys are *not* independently distributed, i.e., the ciphertexts have been calculated using the plaintext and the key.



All characters are elements of a group G.

Plaintext, key and ciphertext are character strings.

For the encryption of a character string *x* of length *n*, a randomly generated and secretly exchanged key $k = (k_1, ..., k_n)$ is used.

The *i*th plaintext character x_i is encrypted as $c_i := x_i + k_i$

It can be decrypted with

$$x_i := c_i - k_i.$$

Evaluation: 1. secure against adaptive attacks2. easy to calculate

3. but key is very long

Keys have to be very long for information-theoretical security

- ${\cal K}\,$ is the set of keys,
- X is the set of plaintexts, and
- C is the set of ciphertexts, which appear at least once.
- $|C| \ge |X|$ otherwise it can't be decrypted (fixed *k*)
- $|\mathcal{K}| \ge |C|$ so that any ciphertext might as well be any plaintext (fixed *x*)
- therefore $|\mathcal{K}| \ge |\mathcal{X}|$.
- If plaintext cleverly coded, it follows that:

The length of the key must be at least the length of the plaintext.



Definition for information-theoretical security

1. Definition for information-theoretical security

(all keys are chosen with the same probability)

 $\forall c \in C. \exists const \in \mathbb{N}. \forall x \in X: |\{k \in K | k(x) = c\}| = const.$ (1)

The a-posteriori probability of the plaintext x is P(x|c), after the attacker got to know the ciphertext c.

2. Definition

$$\forall c \in C. \ \forall x \in X: P(x|c) = P(x).$$

Both definitions are equivalent (if P(x) > 0):

According to Bayes:

$$P(x|c) = \frac{P(x) \cdot P(c|x)}{P(c)}$$

Therefore, (2) is equivalent to

$$\forall c \in C. \ \forall x \in X: P(c|x) = P(c).$$

We show that this is equivalent to

 $\forall c \in C. \exists const' \in \mathbb{R} \ \forall x \in X: \ P(c \mid x) = const'.$

(2)

(3)

(4)



(3) \Rightarrow (4) is clear with *const* := P(c).

(4) \Rightarrow (3): Conversely, we show *const*' = P(c):

$$\sum_{x} P(x) \cdot P(c|x) \qquad \forall c \in C \exists const' \in \mathsf{IR} \ \forall x \in X: \ P(c|x) = const' \quad (4)$$

$$P(c) = \sum_{x} P(x) \cdot const' \qquad \forall c \in C \ \forall x \in X: \ P(c|x) = P(c). \quad (3)$$

$$const' \cdot \sum_{x} P(x) \qquad \forall c \in C \ \exists const \in \mathsf{IN} \ \forall x \in X: |\{k \in K | \ k(x) = c\}| = const. \quad (1)$$

(4) is already quite the same as (1): In general holds

 $P(c|x) = P(\{k \mid k(x) = c\}),$

and if all keys have the same probability,

 $P(c|x) = |\{k \mid k(x) = c\}| / |\mathcal{K}|.$ Then (4) is equivalent (1) with

 $const = const' \bullet |K|.$



Key distribution:

like for symmetric encryption systems

Simple example (view of attacker)

The outcome of tossing a coin (Head (H) or Tail (T)) shall be sent in an authenticated fashion:

Key		m, MAC			
		H,0	H,1	Τ,0	T,1
k	00	Н		Т	
	01	Н			Т
	10		н	Т	
	11		Н		Т

Security: e.g. attacker wants to send T.

- a) blind: get caught with a probability of 0.5
- b) seeing: e.g. attacker gets (H,0) $\implies k \in \{00, 01\}$

still both, (T,0) and (T,1), have a probability of 0.5



- Definition "Information-theoretical security"
- with error probability \mathcal{E} :
 - $\forall x$, MAC (that attacker can see)
 - $\forall y \neq x$ (that attacker sends instead of x)
 - \forall MAC' (where attacker chooses the one with the highest probability fitting y)

 $P(k(y) = MAC' \mid k(x) = MAC) \le \mathcal{E}$

(probability that MAC' is correct if one only takes the keys k which are still possible under the constraint of (x,MAC) being correct.)

Improvement of the example:

a) 2σ key bits instead of 2: $k = k_1 k_1^* \dots k_\sigma k_\sigma^*$ MAC = MAC₁,...,MAC_{σ}; MAC_{*i*} calculated using $k_i k_i^*$

 \Rightarrow error probability 2^{- σ}

b) *l* message bits: $x^{(1)}$, MAC⁽¹⁾ = MAC₁⁽¹⁾, ..., MAC_{σ}⁽¹⁾ \vdots $x^{(l)}$, MAC^(l) = MAC₁^(l), ..., MAC_{σ}^(l)



Limits:

```
\sigma-bit-MAC \Rightarrow error probability \ge 2^{-\sigma} (guess MAC)
```

```
σ
-bit-key ⇒ error probability ≥ 2<sup>-σ</sup>
(guess key, calculate MAC)
```

still clear: for an error probability of $2^{-\sigma}$, a σ -bit-key is too short, because k(x) = MAC eliminates many values of k.

Theorem: for a single/the first message you need 2σ -bit-key

(for succeeding messages σ new key bits suffice, if recipient adequately responds on authentication "errors": attacker learns σ key bits with every message)

Possible at present: $\approx 4\sigma \cdot \log_2(\text{length}(x))$ key bits

```
(Wegman, Carter)
```

much shorter as one-time pad



```
Mathematical secrets:
```

```
(to decrypt, to sign ...)
p, q, prime numbers
```

Public part of key-pair:

(to encrypt, to test ...)

 $n = p \cdot q$

```
p, q big, at present \approx \mathcal{L} = 500 up to 2000 bit (theory : \mathcal{L} \rightarrow \infty)
```

Often: special property

 $p \equiv q \equiv 3 \mod 4$

(the semantics of " \equiv ... mod" is: $a \equiv b \mod c$ iff *c* divides *a*-*b*, putting it another way: dividing *a* and *b* by *c* leaves the same remainder)



application:

s²-mod-*n*-generator,GMR and many others,e.g., only well analyzed systems like RSA

(significant alternative: only "discrete logarithm", based on number theory, too, similarly well analyzed)

necessary: 1. factoring is difficult

- 2. to generate *p*,*q* is easy
- 3. operations on the message with *n* alone, you can only invert using *p*, *q*



Factoring

clear: in NP \Rightarrow but difficulty cannot be proved yet complexity at present

$$L(n) = e^{c \cdot \sqrt[3]{\ln(n) \cdot (\ln \ln(n))^2}}, c \approx 1,9$$

$$\approx e^{\sqrt[3]{l}}$$
 "sub-exponential"

practically up to 155 decimal digits in the year 1999 174 decimal digits in the year 2003 200 decimal digits in the year 2005 232 decimal digits in the year 2010 240 decimal digits in the year 2019 (www.crypto-world.com/FactorRecords.html) 250 decimal digits in the year 2020

(notice :

 \exists faster algorithms, e.g., for $2^r \pm 1$, but this doesn't matter)

assumption: factoring is hard (notice : unacceptable, if attacker could factor, e.g., every $1000^{\text{th}} n$)



- \forall PPA \mathcal{F} (probabilistic polynomial algorithm, which tries to factor)
- $\forall \text{ polynomials } \mathcal{Q}$
- $\exists L \forall \ell \geq L$: (asymptotically holds:)

If *p*, *q* are random prime numbers of length \mathcal{L} and $n = p \cdot q$:

$$P(\mathcal{F}(n) = (p, q)) \leq \frac{1}{\mathcal{Q}(\mathcal{L})}$$

(probability that \mathcal{F} truly factors decreases faster as $\frac{1}{\text{any polynomial}}$.)

trustworthy ??

the best analyzed assumption of all available



1. Are there enough prime numbers ? (important also for factoring assumption)

 $\frac{\pi(x)}{x} \approx \frac{1}{\ln(x)}$ $\pi(x) \text{ number of the prime numbers} \leq x$ "prime number theorem" $\Rightarrow \text{ up to length } \mathcal{L} \text{ more than every } \mathcal{L}^{\text{th}}.$ And $\approx \text{ every } 2^{\text{nd}} \equiv 3 \mod 4$ "Dirichlet's prime number theorem"

2. Principle of search:

repeat

choose random number $p (\equiv 3 \mod 4)$ test whether p is prime until p prime



3. Primality tests:

(notice: trying to factor is much too slow) probabilistic; "Rabin-Miller" special case $p \equiv 3 \mod 4$: Little Theorem of Fermat: $a^{p-1} \equiv 1 \mod p$

p prime $\Rightarrow \forall a \neq 0 \mod p : a^{\frac{p-1}{2}} \equiv \pm 1 \pmod{p}$

$$p \text{ not prime} \implies \text{for} \le \frac{1}{4} \text{ of } a's : a^{\frac{p-1}{2}} \equiv \pm 1 \pmod{p}$$

⇒ test this for *m* different, independently chosen values of \mathcal{A} , error probability $\leq \frac{1}{4^m}$ (doesn't matter in general)



Calculating with and without p,q(1)

 Z_n : ring of residue classes mod $n \stackrel{\circ}{=} \{0, ..., n-1\}$

- +, -, fast
- exponentiation "fast" (square & multiply)

example: $7^{26} = 7^{(11010)_2}$; from left $71 \xrightarrow{s} 710$ $7110 \xrightarrow{s} 71100$ 711010 711010



• gcd (greatest common divisor) fast in Z (Euclidean Algorithm)



 Z_n^* : multiplicative group $a \in Z_n^* \Leftrightarrow \text{gcd}(a,n) = 1$

Inverting is fast (extended Euclidean Algorithm)
 Determine to *a*,*n* the values *u*,*v* with

 $a \cdot u + n \cdot v = 1$

Then: $u \equiv a^{-1} \mod n$

example: $3^{-1} \mod 11$? $= -11 + 4 \cdot 3$ $11 = 3 \cdot 3 + 2$ $3 = 1 \cdot 2 + 1$ $1 = 1 \cdot 3 - 1 \cdot (11 - 3 \cdot 3)$ $1 = 1 \cdot 3 - 1 \cdot 2$

 \Rightarrow 3⁻¹ \equiv 4 mod 11



Number of elements of Z_n^*

The Euler Φ - Function is defined as

$$\Phi(n) := |\{a \in \{0,...,n-1\} | gcd(a,n)=1\}|,$$

whereby for any integer $n \neq 0$ holds: gcd (0,n) = |n|.

It immediately follows from both definitions, that

$$Z_n^{*} = \Phi(n).$$

For $n = p \bullet q$, p,q prime and $p \neq q$ we can easily calculate $\Phi(n)$: $\Phi(n) = (p-1) \bullet (q-1)$

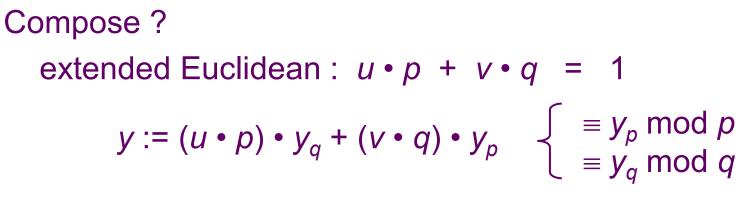
gcd \neq 1 have the numbers 0, then *p*, 2*p*, ..., (*q*-1)*p* and *q*, 2*q*, ..., (*p*-1)*q*, and these 1+(*q*-1)+(*p*-1) = *p*+*q*-1 numbers are for *p* \neq *q* all different.



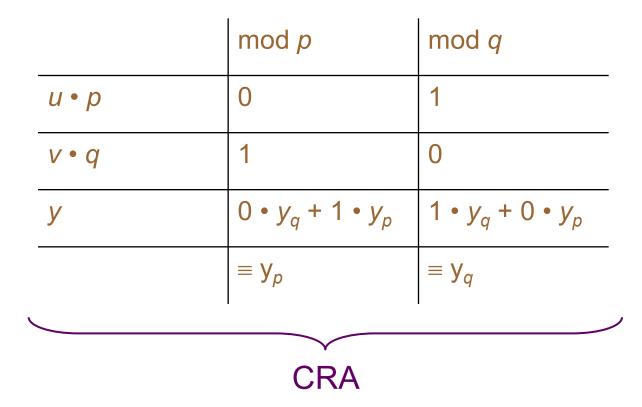
 \Rightarrow To calculate f(x) mod n, at first you have to calculate mod p, q separately.

 $y_p := f(x) \mod p$ $y_q := f(x) \mod q$





Since :





R. Rivest, A. Shamir, L. Adleman: A Method for obtaining Digital Signatures and Public-Key Cryptosystems; Communications of the ACM 21/2 (Feb. 1978) 120-126.

Key generation

- 1) Choose two prime numbers p and q at random as well as stochastically independent, with $|p| \approx |q| = \mathcal{U}$, $p \neq q$
- 2) Calculate $n := p \cdot q$
- 3) Choose c with $3 \le c \le (p-1)(q-1)$ and

4) Calculate *d* using *p*, *q*, *c* as multiplicative inverse of *c* mod $\Phi(n)$ $c \cdot d \equiv 1 \pmod{\Phi(n)}$

5) Publish *c* and *n*.

En- / decryption

exponentiation with c respectively d in Z_n

Proposition: $\forall m \in \mathbb{Z}_n$ holds: $(m^c)^d \equiv m^{c \cdot d} \equiv (m^d)^c \equiv m \pmod{n}$



$$c \cdot d \equiv 1 \pmod{\Phi(n)} \Leftrightarrow$$

$$\exists k \in \mathbb{Z} : c \cdot d - 1 = k \cdot \Phi(n) \Leftrightarrow$$

$$\exists k \in \mathbb{Z} : c \cdot d = k \cdot \Phi(n) + 1$$

Therefore $m^{c \cdot d} \equiv m^{k \cdot \Phi(n) + 1} \pmod{n}$
Using the Theorem of Euler/Fermat
 $\forall m \in \mathbb{Z}_n^* : m^{\Phi(n)} \equiv 1 \pmod{n}$

it follows for all *m* coprime to *p* $m^{p-1} \equiv 1 \pmod{p}$ [Little Theorem of Fermat]

Because *p*-1 is a factor of $\Phi(n)$, it holds

$$m^{k \cdot \Phi(n)+1} \equiv_{p} m^{k \cdot (p-1)(q-1)+1} \equiv_{p} m \cdot (\underbrace{m^{p-1}}_{1})^{k \cdot (q-1)} \equiv_{p} m$$



Holds, of course, for $m \equiv_p 0$. So we have it for all $m \in Z_p$. Same argumentation for q gives

$$m^{k \bullet \Phi(n) + 1} \equiv_q m$$

Because congruence holds relating to p as well as q, according

to the CRA, it holds relating to $p \cdot q = n$.

Therefore, for all $m \in Z_n$

$$m^{c \cdot d} \equiv m^{k \cdot \Phi(n) + 1} \equiv m \pmod{n}$$

Attention:

There is (until now ?) **no** proof RSA is easy to break \Rightarrow to factor is easy

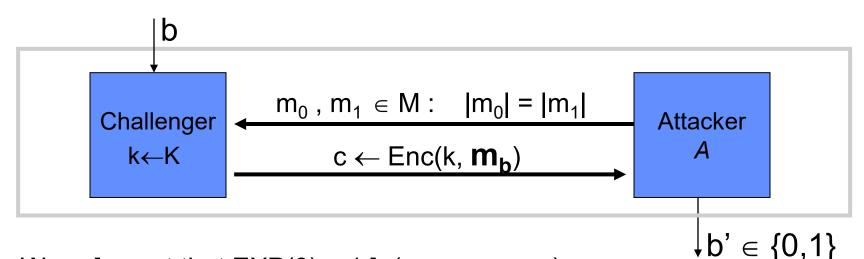


Semantic Security



Let's play a game:

A challenger flips a coin, and the adversary guesses the outcome For b=0,1 define experiments EXP(0) and EXP(1) as:

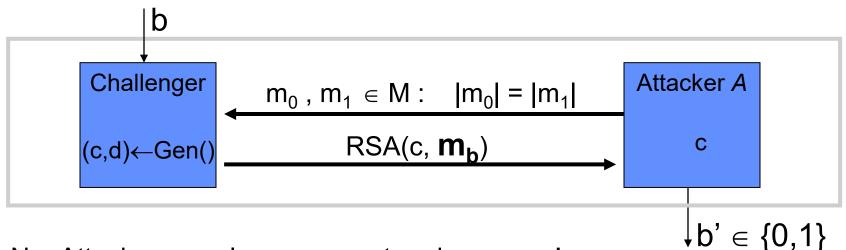


- $W_0 := [$ event that EXP(0) = 1] (wrong guess)
- W₁ := [event that EXP(1) = 1] (correct guess)
- $Advantage_{SS}[A, Enc] \coloneqq |\Pr[W_0] \Pr[W_1]| \in [0,1]$
- Enc is called semantically secure if for all efficient algorithms A, *Advantage_{SS}*[A, Enc] is negligible (~0)



Let's play a game:

A challenger flips a coin, and the adversary guesses the outcome For b=0,1 define experiments EXP(0) and EXP(1) as:



No: Attacker can always encrypt and compare!

➔ indeterministic encryption necessary! add random number...

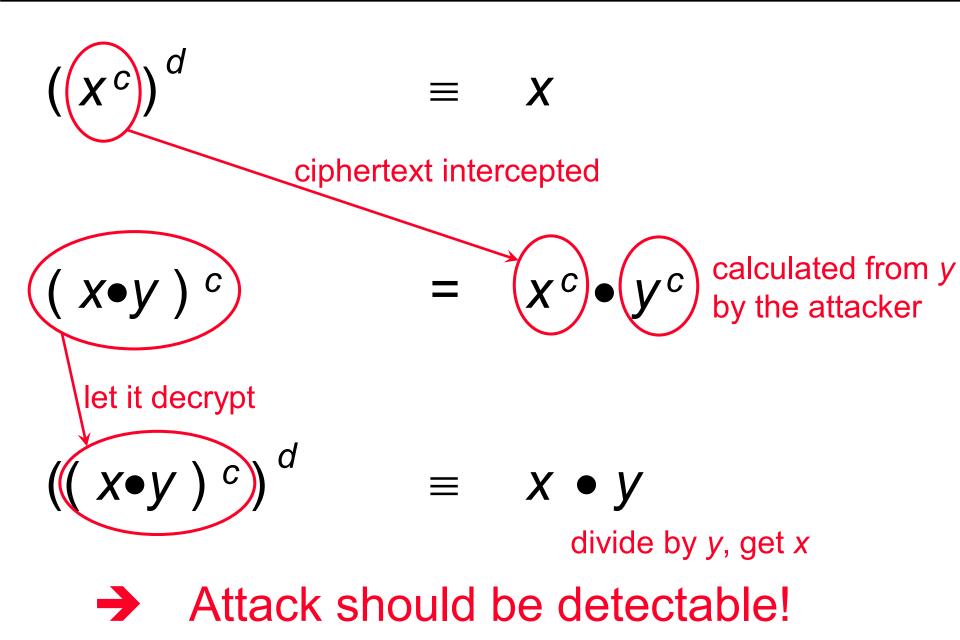


$$(x^{c})^{d} \equiv X$$
 Encryption/Decryption
Homomorphic Property of RSA:
 $Enc(m_{1}) \cdot Enc(m_{2}) = Enc(m_{1} \cdot m_{2})$
 $(x \bullet y)^{c} = x^{c} \bullet y^{c}$

 $((X \bullet Y)^{c})^{d} \equiv X \bullet Y$ Encryption/Decryption



Attack on encryption with RSA naive





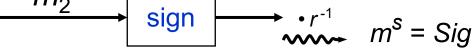
Attack on digital signature with RSA naive

1. Simple version of Davida's attack:

Given $Sig_1 = m_1^s$ $Sig_2 = m_2^s$ \Rightarrow $Sig := Sig_1 \cdot Sig_2 = (m_1 \cdot m_2)^s$ New signature generated ! (Passive attack, *m* not selectable.)

2. Active, desired $Sig = m^s$ Choose any m_1 ; $m_2 := m \cdot m_1^{-1}$ Let m_1 , m_2 be signed. Further as mentioned above.

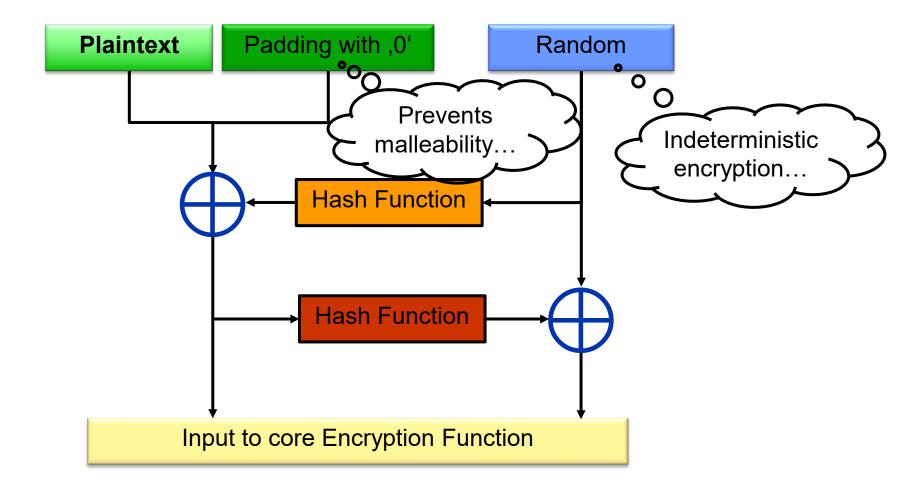
3. Active, more skillful (Moore) "Blinding": choose any r, $m_2 := m \cdot r^t$ $m_2^s = m^s \cdot r^{t \cdot s} = m^s \cdot r$ m_2



Defense against Davida's attacks using a collision-resistant hash function

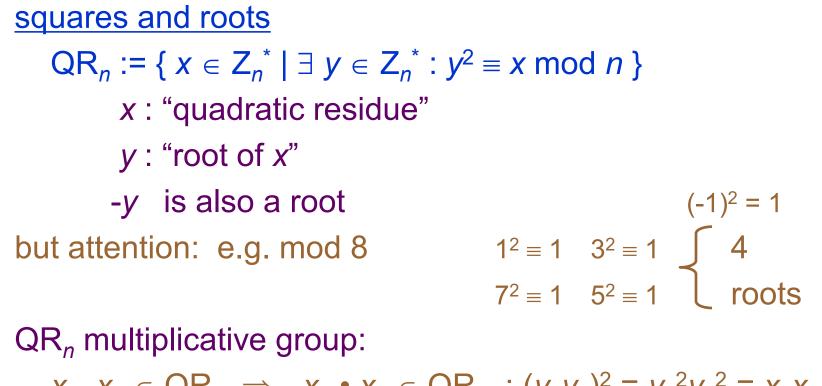
h(): collision-resistant hash function

Before signing, h is applied to the message signature of $m = (h(m))^s \mod n$ test if $h(m) = ((h(m))^s)^t \mod n$



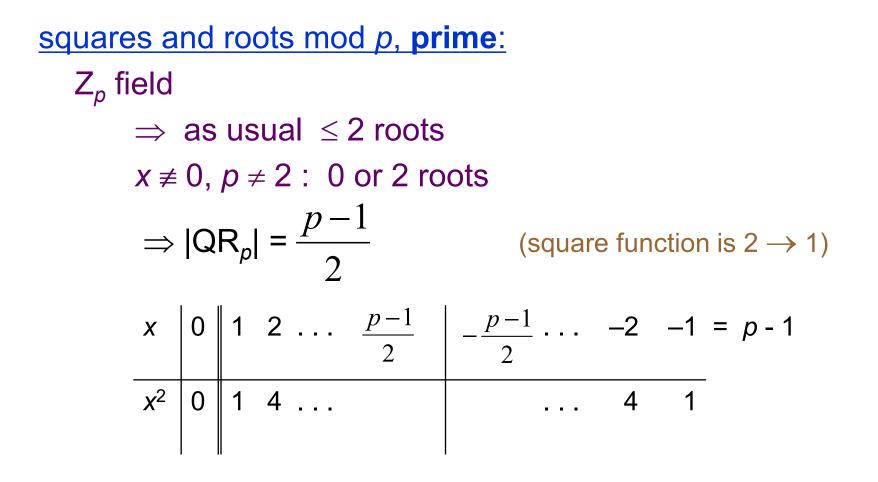
Semantic security against adaptive chosen ciphertext attacks





 $\begin{array}{rcl} x_1, \, x_2 \in {\rm QR}_n \implies & x_1 \bullet x_2 \in {\rm QR}_n \ : \, (y_1 y_2)^2 = y_1^2 y_2^2 = x_1 x_2 \\ & x_1^{-1} & \in {\rm QR}_n \ : \, (y_1^{-1})^2 \ = (y_1^2)^{-1} = x_1^{-1} \end{array}$





Jacobi symbol

$$\left[\frac{x}{p}\right] := \begin{cases} 1 & \text{if } x \in QR_p \\ -1 & \text{else} \end{cases} \quad (\text{for } x \in Z_p^*)$$



Continuation squares and roots mod p, prime:

Euler criterion : $\left(\frac{x}{p}\right) \equiv x^{\frac{p-1}{2}} \mod p$

(i.e. fast algorithm to test whether square)

Proof using little Theorem of Fermat: $x^{p-1} \equiv 1 \mod p$ co-domain ok : $x^{\frac{p-1}{2}} \in \{\pm 1\}$, because $\left(x^{\frac{p-1}{2}}\right)^2 \equiv 1$ x square : $\left[\frac{x}{p}\right] = 1 \Rightarrow x^{\frac{p-1}{2}} \equiv (y^2)^{\frac{p-1}{2}} \equiv y^{p-1} \equiv 1$ x nonsquare : The $\frac{p-1}{2}$ solutions of $x^{\frac{p-1}{2}} \equiv 1$ are the squares.

So no nonsquare satisfies the equation.

Therefore:
$$x^{\frac{p-1}{2}} \equiv -1$$

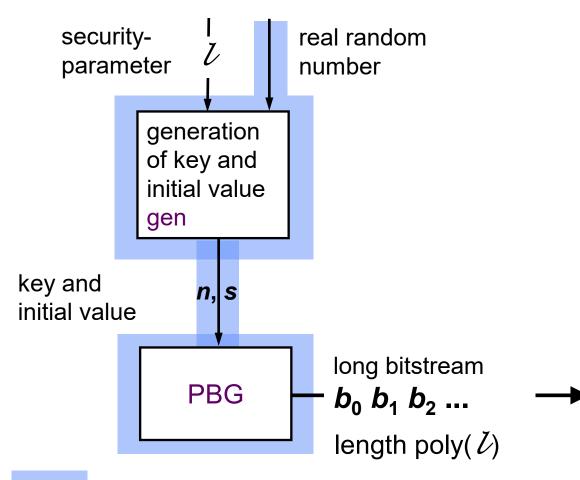


Idea: short initial value (seed) \rightarrow long bit sequence (should look random from a polynomial attacker's point of view)



Scheme:

Idea: short initial value (seed) \rightarrow long bit sequence (should look random from a polynomial attacker's point of view)

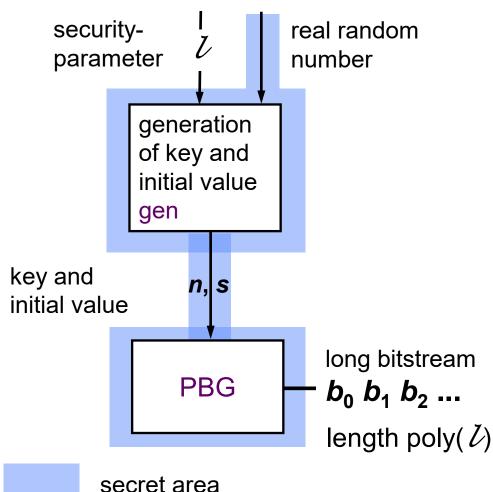


secret area



Idea: short initial value (seed) \rightarrow long bit sequence (should look random from a polynomial attacker's point of view)





Requirements:

- gen and PBG are efficient
- PBG is deterministic
 - $(\Rightarrow$ sequence reproducible)
- secure: no probabilistic polynomial test can distinguish PBG-streams from real random streams



Method

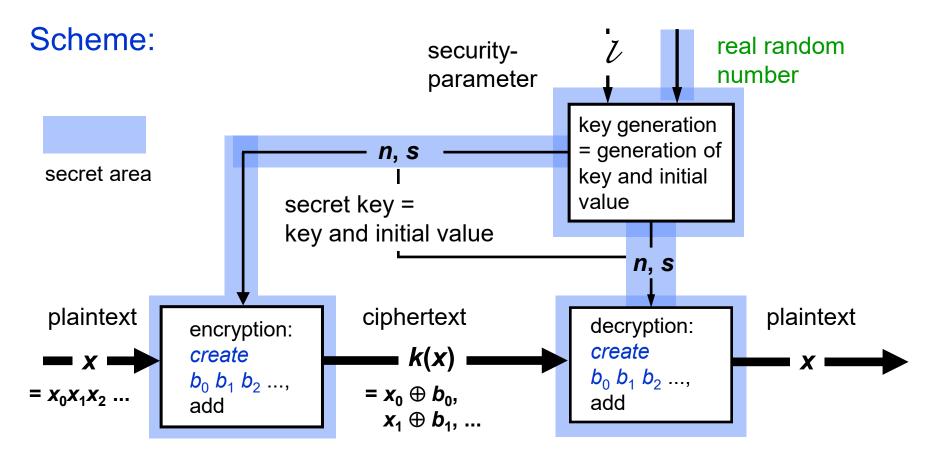
 key value: 	p,q : prime / big / \equiv 3 mo $n = p \cdot q$	d 4
initial value (seed):PBG:	$s \in Z_n^*$ $s_0 := s^2 \mod n$	
	$s_{i+1} := s_i^2 \mod n$	$b_i := s_i \mod 2$ (last bit)
Example: $n = 3 \cdot 11 = 3$ index	$\begin{array}{c} 3, \ \mathbf{s} = 2 \\ 0 \ 1 \ 2 \ 3 \ 4 \end{array}$	16² mod 33 = 8 ⋅ 32 = 8 ⋅ (-1) = 25
s _i : b _i :	4162531400110	$25^{2} = (-8)^{2} \equiv 64 \equiv 31$ $31^{2} = (-2)^{2} = 4$

Note: length of period no problem with big numbers



s²-mod-*n*-generator as symmetric encryption system

- Purpose: application as symmetric encryption system: "Pseudo one-time pad"
- Compare: one-time pad: add long real random bit stream with plaintext Pseudo one-time pad: add long pseudo-random stream with plaintext





Idea:

If no probabilistic polynomial test can distinguish pseudo-random streams from real random streams, then the pseudo one-time pad is as good as the one-time pad against polynomial attacker.

(Else the attack is a test !)

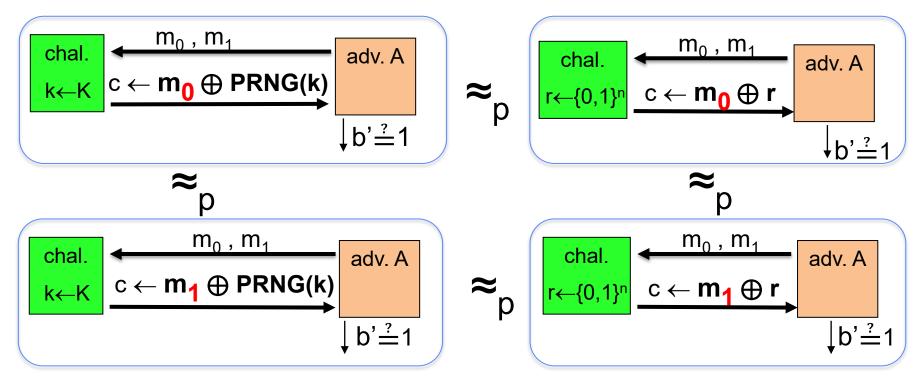
Construction works with any good PBG

Proof of security of pseudo one-time pad: another approach

• Prerequisite:

(Based on slide from Prof. Thorsten Strufe)

- Unpredictable PRNG, which cannot be distinguished from real randomness
- We known:
 - One-time pad (XOR with truly random bit string) is secure
- Proof intuition:





squares and roots mod $p \equiv 3 \mod 4$

• extracting roots is easy: given $x \in QR_p$ $w \coloneqq x^{\frac{p+1}{4}} \mod p$ is root proof : 1. $p \equiv 3 \mod 4 \Rightarrow \frac{p+1}{4} \in \mathbb{N}$ 2. $w^2 = x^{\frac{p+1}{2}} = x^{\frac{p-1}{2}+1} = x^{\frac{p-1}{2}} \cdot x = 1 \cdot x$ Euler, $x \in QR_p$ In addition: $w \in QR_p$ (power of $x \in QR_p$) \rightarrow extracting roots iteratively is possible • $\left[\frac{-1}{p}\right] \equiv (-1)^{\frac{p-1}{2}} \equiv (-1)^{\frac{4r+2}{2}} \equiv (-1)^{\frac{2r+1}{2}} = (-1)^{\frac{2r+1}{2}} = -1$

$$p = 4r+3$$

 \Rightarrow -1 \notin QR_p

⇒ of the roots ± w: $-w \notin QR_p$ (otherwise $-1 = (-w) \cdot w^{-1} \in QR_p$)

squares and roots mod *n* <u>using</u> *p*,*q* (usable as secret operations)

• testing whether square is simple $(n = p \cdot q, p, q \text{ prime}, p \neq q)$ $x \in QR_n \iff x \in QR_p \land x \in QR_q$ Chinese Remainder Theorem proof: " \Rightarrow " $x \equiv w^2 \mod n \Rightarrow x \equiv w^2 \mod p \land x \equiv w^2 \mod q$ " \Leftarrow " $x \equiv w_p^2 \mod p \land x \equiv w_q^2 \mod q$ $w := CRA(w_{D}, w_{d})$ then $w \equiv w_p \mod p \land w \equiv w_q \mod q$ using the Chinese Remainder Theorem for $w^2 \equiv w_p^2 \equiv x \mod p \land w^2 \equiv w_q^2 \equiv x \mod q$ we have $w^2 \equiv x \mod n$



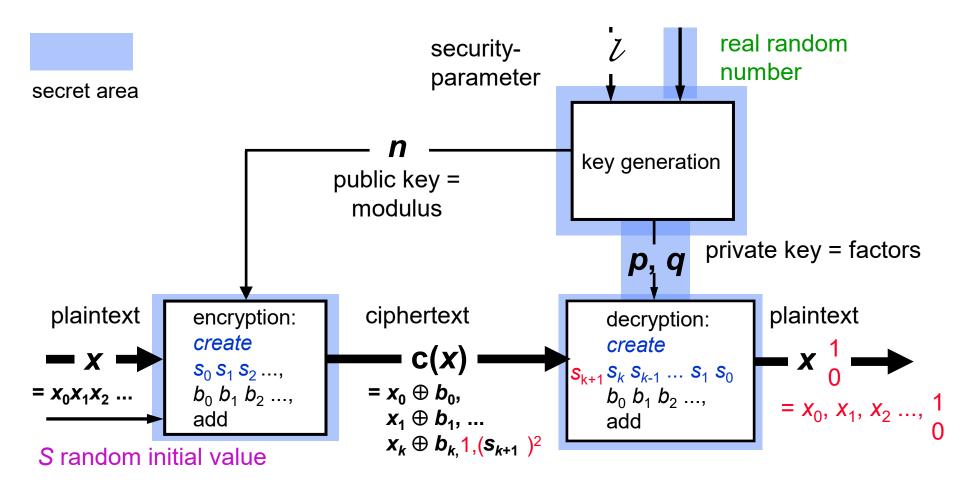
Continuation squares und roots mod *n* using *p*,*q*

- $x \in QR_n \Rightarrow x$ has exactly 4 roots (mod *p* and mod $q : \pm w_p, \pm w_q$. therefore the 4 combinations according to the Chinese Remainder Theorem)
- extracting a root is easy (p, q ≡ 3 mod 4) determine roots w_p, w_q mod p, q

$$w_p \coloneqq x^{\frac{p+1}{4}} \qquad \qquad w_q \coloneqq x^{\frac{q+1}{4}}$$

combine using CRA

chosen ciphertext-plaintext attack





Continuation squares und roots mod n using p,q

Jacobi symbol

$$\left[\frac{x}{n}\right] := \left[\frac{x}{p}\right] \bullet \left[\frac{x}{q}\right]$$

So:
$$\left(\frac{x}{n}\right) = \begin{cases} +1 & \text{if} \\ -1 & \text{if} \end{cases}$$

 $x \in QR_p \land x \in QR_q \lor$ $x \notin QR_p \land x \notin QR_q$ "cross-over"

So : $x \in QR_n$

$$\Rightarrow \left[\frac{x}{n}\right] = 1$$

$$\notin \text{ does not hold}$$



continuation squares und roots mod *n* using *p*,*q*

to determine the Jacobi symbol is easy

e.g. $p \equiv q \equiv 3 \mod 4$ $\left(\frac{-1}{n}\right) = \left(\frac{-1}{p}\right) \bullet \left(\frac{-1}{q}\right) = (-1)\bullet(-1) = 1$

but $-1 \notin QR_n$, because $-1 \notin QR_{p,q}$



squares and roots mod *n* without *p*,*q*

- extracting roots is difficult: provably so difficult as to factor
 - a) If someone knows 2 significantly different roots of an *x* mod *n*, then he can definitely factor *n*.

(i.e. $w_1^2 \equiv w_2^2 \equiv x$, but $w_1 \not\equiv \pm w_2 \Rightarrow n \not\mid (w_1 \pm w_2)$) proof: $n \mid w_1^2 - w_2^2 \Rightarrow n \mid (w_1 + w_2)(w_1 - w_2)$

p in one factor, q in the other

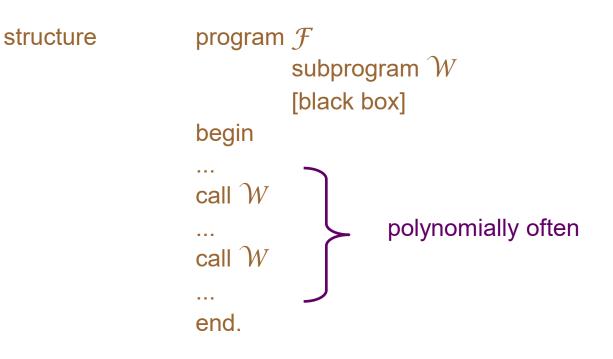
 \Rightarrow gcd($w_1 + w_2, n$) is p or q



Calculating with and without *p*,*q* (15)

Continuation squares und roots mod *n* without *p*,*q*

b) Sketch of "factoring is difficult \Rightarrow extracting a root is difficult" proof of "factoring is easy \Leftarrow extracting a root is easy" So assumption : $\exists W \in PPA$: algorithm extracting a root to show : $\exists \mathcal{F} \in PPA$: factoring algorithm





to b)

 \mathcal{F} : input *n*

repeat forever

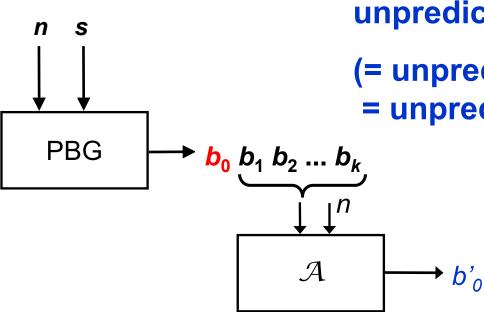
choose $w \in Z_n^*$ at random, set $x := w^2$ $w' := \mathcal{W}(n,x)$ test whether $w' \neq \pm w$, if so factor according to a) break

to determine the Jacobi symbol is easy
 (if *p* and *q* unknown: use quadratic law of reciprocity)

but note : If $\left\lfloor \frac{x}{n} \right\rfloor = 1$, determine whether $x \in QR_n$ is difficult

(i.e. it does not work essentially better than to guess)

QRA = quadratic residuosity assumption



unpredictability to the left will do

(= unpredictability to right = unpredictability of middle)

(see L. Blum, M. Blum, M. Shub 1986)

*s*²-mod-*n*-generator is cryptographically strong: \Leftrightarrow

 $\forall \ \mathcal{A} \in \mathsf{PPA} \qquad \{ \text{ predictor for } \boldsymbol{b}_0 \}$

 \forall constants δ , $0 < \delta < 1$ { frequency of "bad" *n* }

 $\forall t \in N$: { degree of the polynomial }

if $\mathcal{L}(=|n|)$ sufficiently big it holds: for all keys *n* except of at most a δ -fraction

$$\mathsf{P}(\boldsymbol{b}_0 = \mathcal{A}(n, \boldsymbol{b}_1 \boldsymbol{b}_2 \dots \boldsymbol{b}_k) = \mathbf{b'}_0 | \mathbf{s} \in \mathbf{Z}_n^* \text{ random}) \leftarrow \frac{1}{2} + \frac{1}{l^t}$$

Security of the s²-mod-*n*-generator (2)

Proof: Contradiction to QRA in 2 steps

- Assumption: s^2 -mod-*n*-generator is weak, i.e. there is a predictor \mathcal{P} , which guesses b_0 with ε -advantage given $b_1 \ b_2 \ b_3 \dots$
- Step 1: Transform \mathcal{P} in \mathcal{P}^* , which to a given s_1 of QR_n guesses the last bit of s_0 with ε -advantage.

Given s_1 .

Generate $b_1 b_2 b_3 \dots$ with s^2 -mod-*n*-generator, apply \mathcal{P} to that stream. \mathcal{P} guesses b_0 with ε -advantage. That is exactly the result of \mathcal{P}^* .

Step 2: Construct using \mathcal{P}^* a method \mathcal{R} , that guesses with ε -advantage, whether a given s^* with Jacobi symbol +1 is a square.

Given s^* . Set $s_1 := (s^*)^2$. Apply \mathcal{P}^* to s_1 . \mathcal{P}^* guesses the last bit of s_0 with ε -advantage, where s^* and s_0 are roots of s_1 ; $s_0 \in QR_n$. Therefore $s^* \in QR_n \iff s^* = s_0$



Security of the *s*²-mod-*n*-generator (3)

The last bit b^* of s^* and the guessed b_0 of s_0 suffice to guess correctly, because

1) if
$$s^* = s_0$$
, then $b^* = b_0$

2) to show: if
$$s^* \neq s_0$$
, then $b^* \neq b_0$

if $s^* \neq s_0$ because of the same Jacobi symbols, it holds $s^* \equiv -s_0 \mod n$ therefore $s^* \equiv n - s_0$ in Z

n is odd, therefore s^* and s_0 have different last bits

The constructed \mathcal{R} is in contradiction to QRA.

Notes:

- 1) You can take $O(\log(\mathcal{L}))$ random bits in place of (last) 1 bit per squaring.
- There is a more difficult proof that s²-mod-*n*-generator is secure under the factoring assumption.



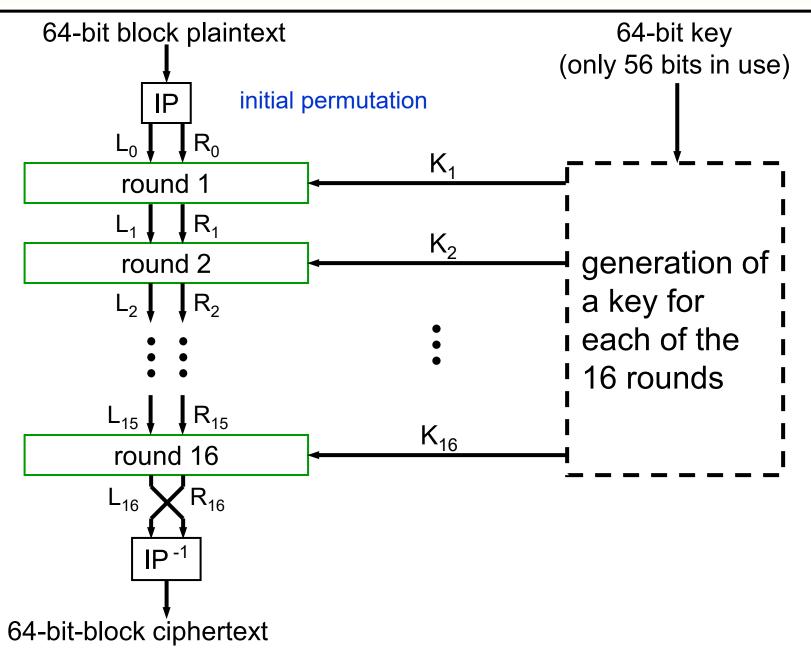
RSA: Faster calculation of the secret operation

 $y^d \equiv w \mod n$ $d_p := c^{-1} \mod p - 1 \implies (y^{d_p})^c \equiv y \mod p$ once and $d_q := c^{-1} \mod q - 1 \implies (y^{d_q})^c \equiv y \mod q$ for all: set $w := CRA(y^{d_p}, y^{d_q})$ every time: $\Rightarrow W^{c} \equiv \begin{cases} (y^{d_{p}})^{c} \equiv y \mod p \\ (y^{d_{q}})^{c} \equiv y \mod q \\ \Rightarrow W^{c} \equiv y \mod n \end{cases}$ proof: How much faster? complexity exponentiation: $\approx \mathcal{L}^3$ complexity 2 exponentiations of half the length: $\approx 2 \cdot \left[\frac{l}{2}\right]^3 = \frac{l^3}{4}$ complexity CRA: 2 multiplications $\approx 2 \bullet \mathcal{L}^2$ 1 addition $\approx \mathcal{L}$

So: \approx Factor 4

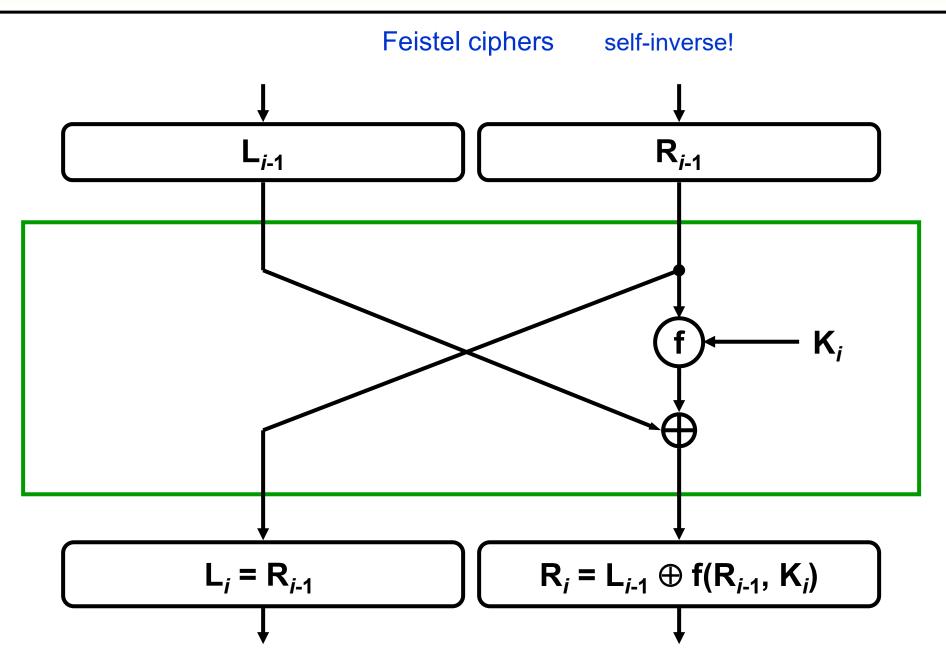
irrelevant





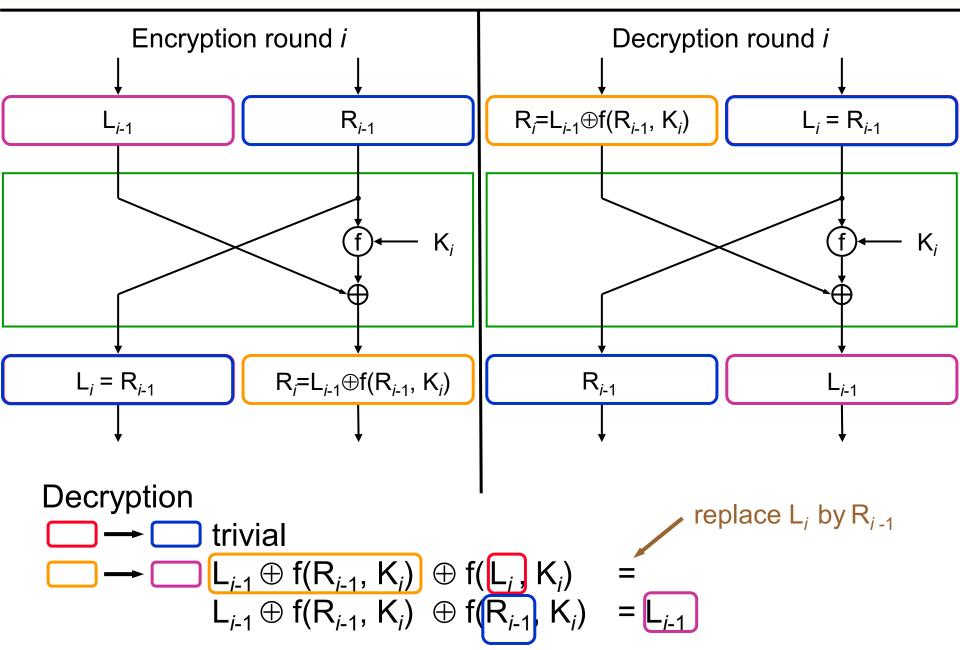


One round

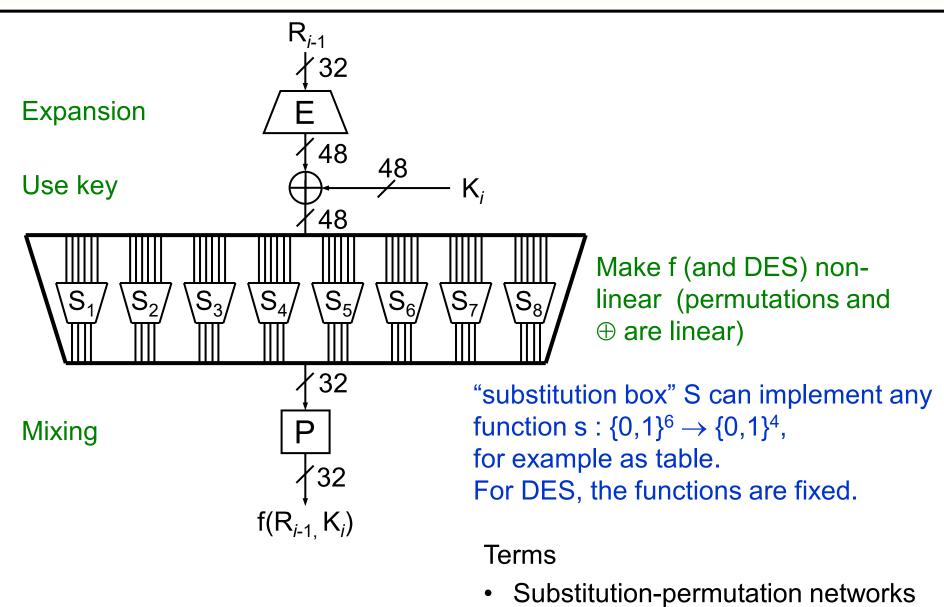




Why does decryption work?

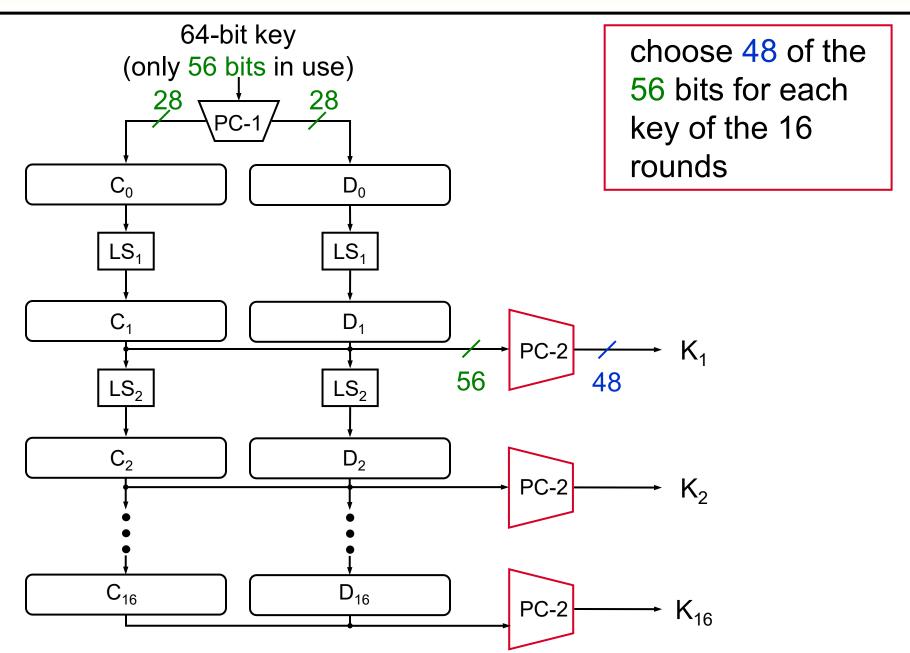






Confusion - diffusion

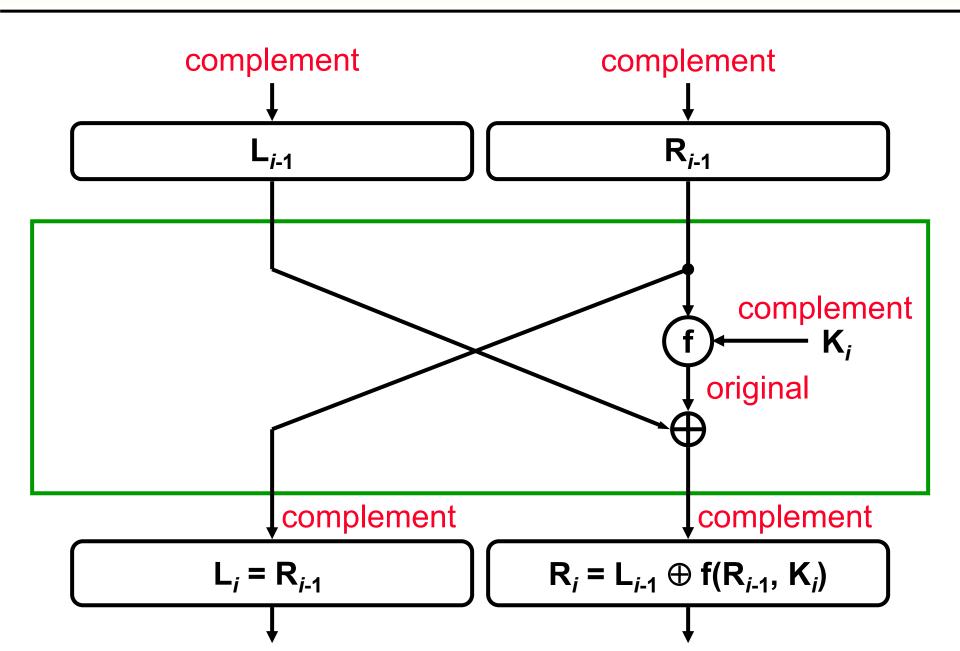




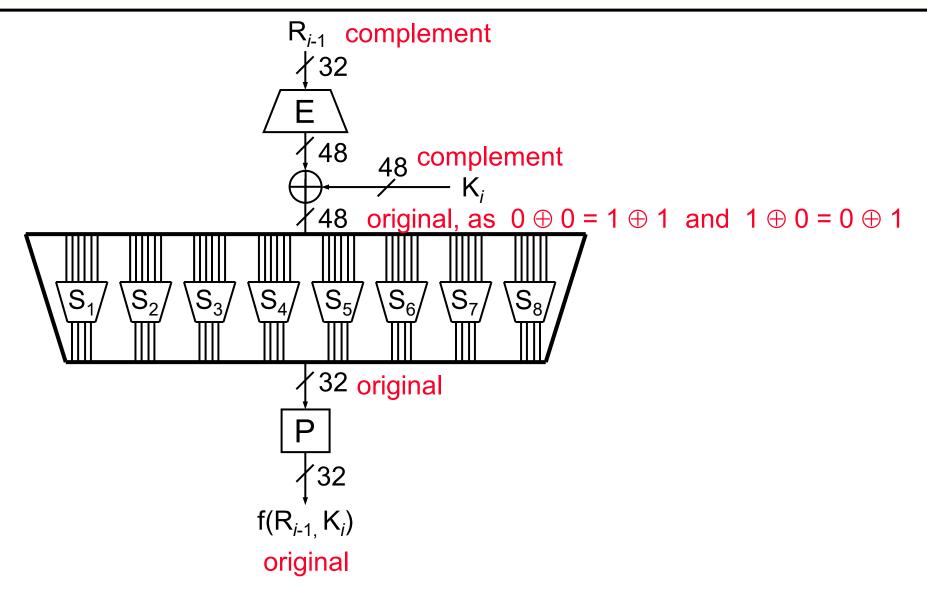


$DES(\overline{k}, \overline{x}) = \overline{DES}(k, x)$











Goal:

Strengthen DES by increasing key length

Let $E: K \times M \longrightarrow M$ be a block cipher (DES)

Define **3E**: $K^3 \times M \longrightarrow M$ as

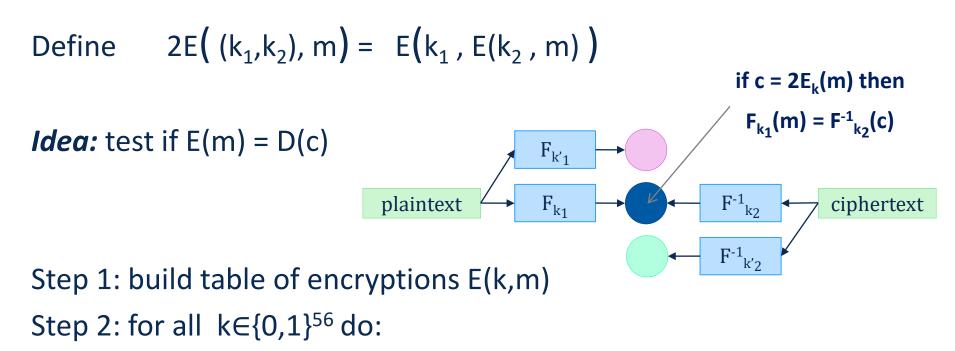
3E($(k_1,k_2,k_3), m$) = E($k_1, D(k_2, E(k_3,m))$)

For 3DES: key-size = $3 \times 56 = 168$ bits. 3×168 slower than DES.

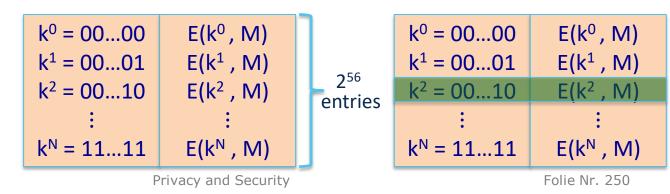
Why not E(E(E(m)))? ... What if: $k_1 = k_2 = k_3$? Simple attack feasible in time $\approx 2^{118}$

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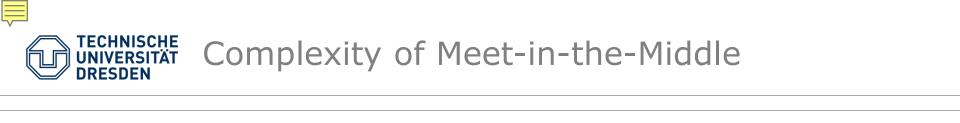




test if D(k, c) is in 2nd column.



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Time = $2^{56}\log(2^{56}) + 2^{56}\log(2^{56}) < 2^{63} << 2^{112}$, space $\approx 2^{56}$

Same attack on 3DES: Time = 2^{118} , space $\approx 2^{56}$



Stream cipher

synchronous self synchronizing

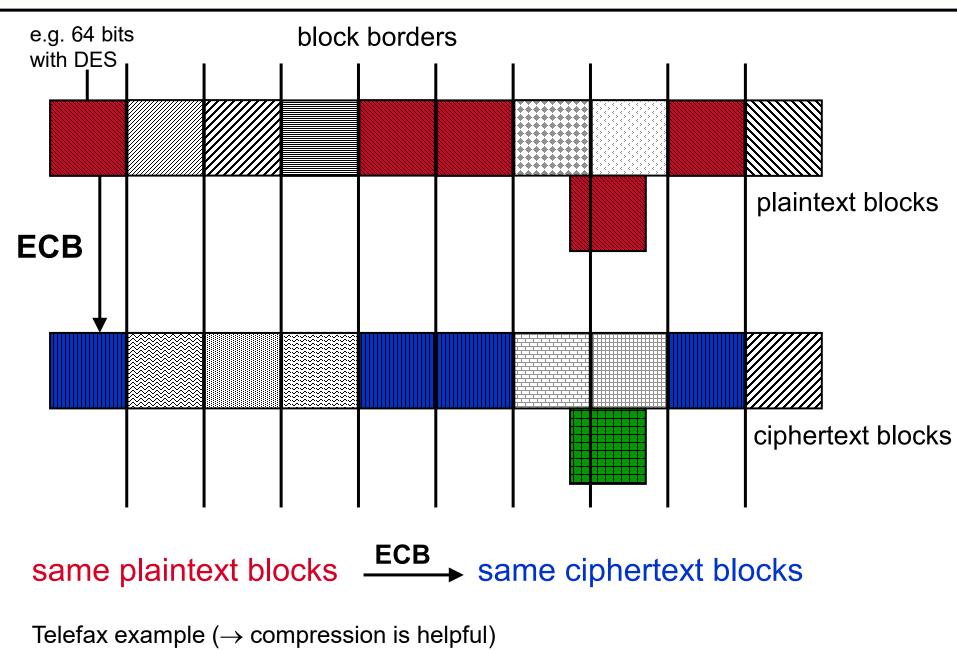
Block cipher

Modes of operation:Simplest:ECB (electronic codebook)each block separatelyBut:concealment: block patterns identifiableauthentication: blocks permutable

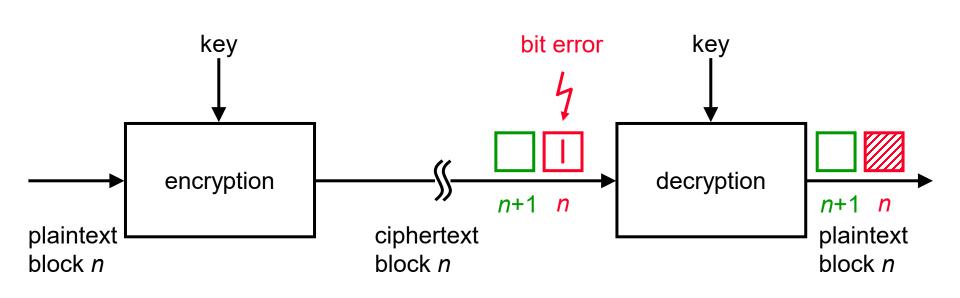


Main problem of ECB

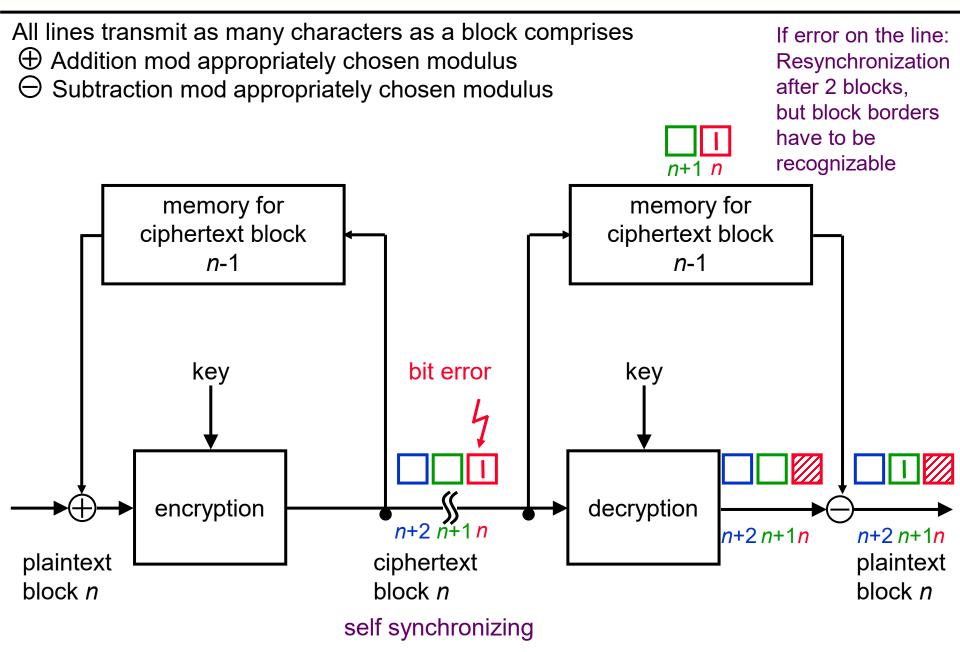
253



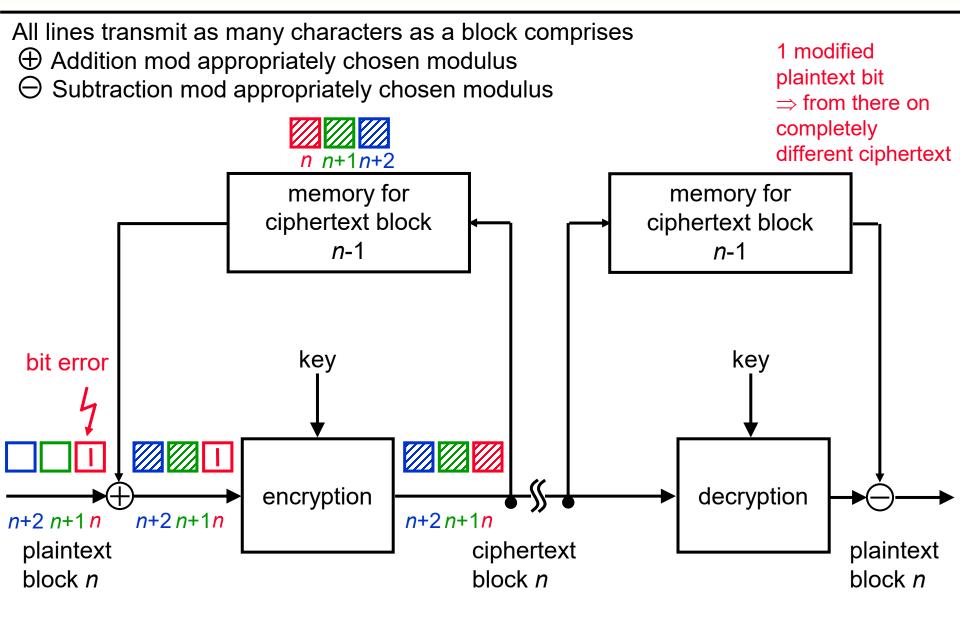






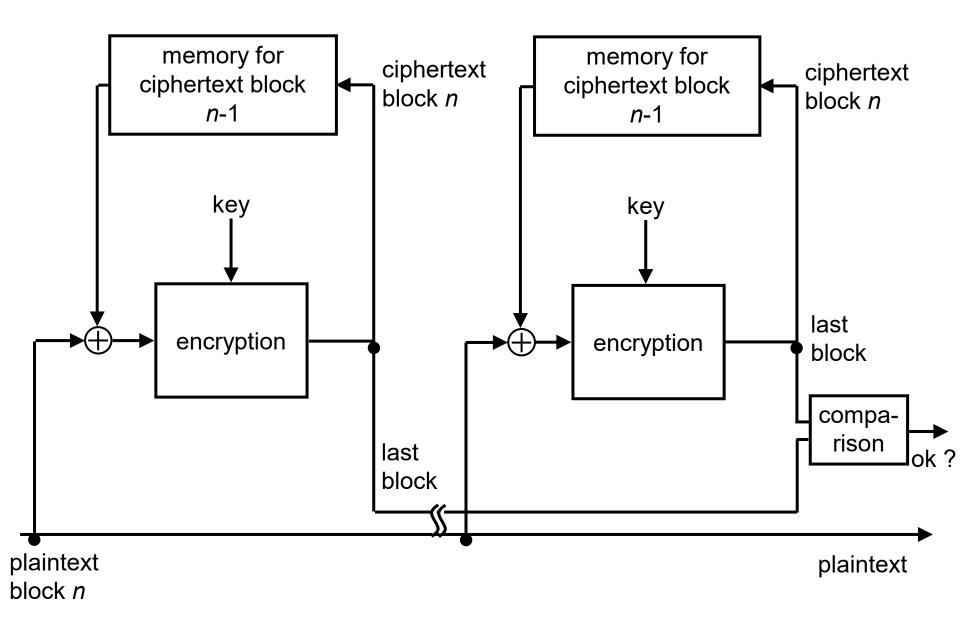


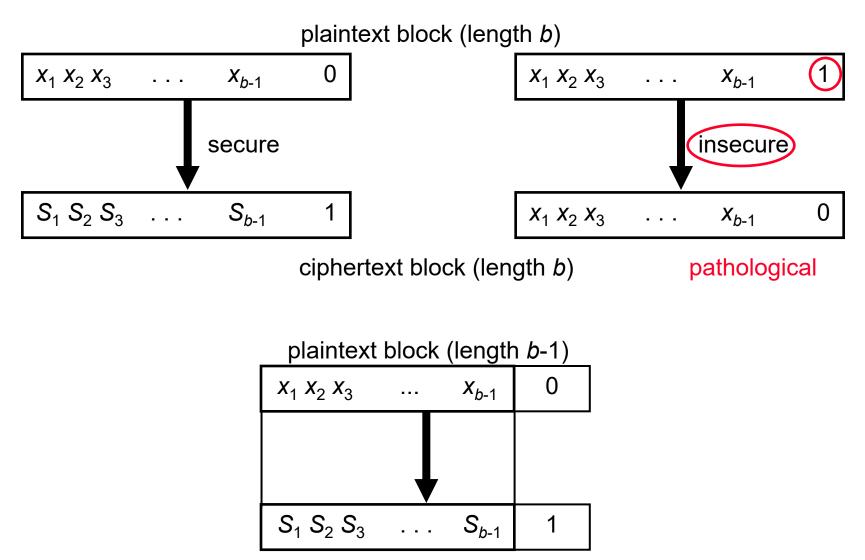




useable for authentication \Rightarrow use last block as MAC





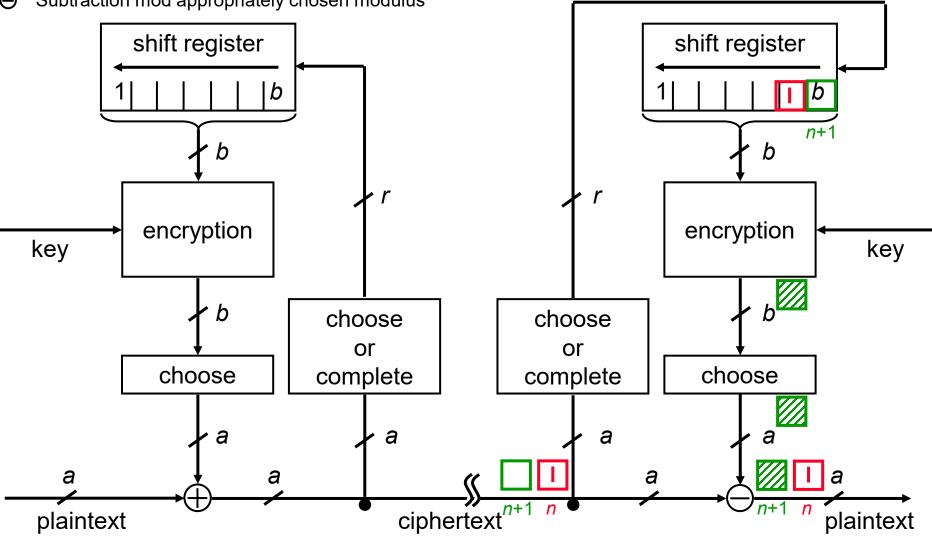


ciphertext block (length b-1)



- Block length b
- Length of the output unit, $a \le b$ а
- Length of the feedback unit, $r \le b$ r
- Addition mod appropriately chosen modulus \oplus
- Subtraction mod appropriately chosen modulus Θ



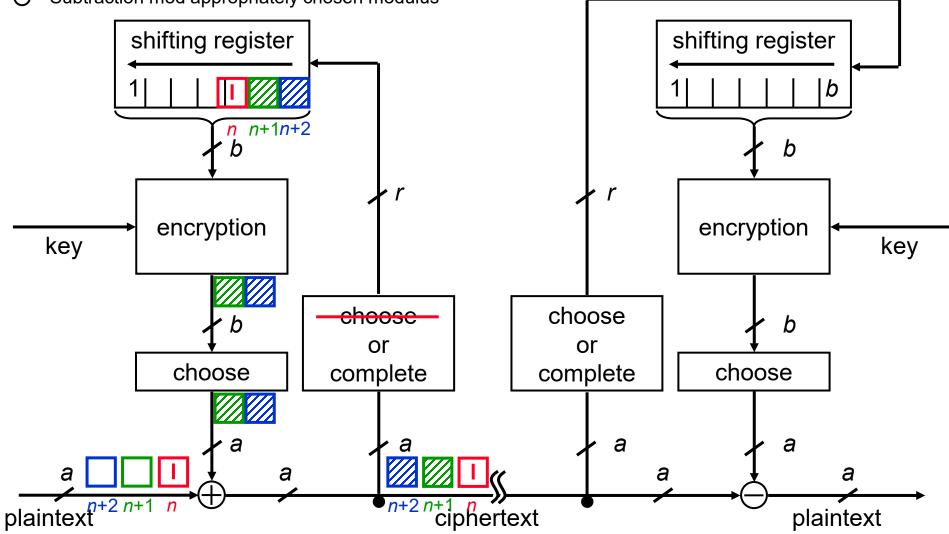




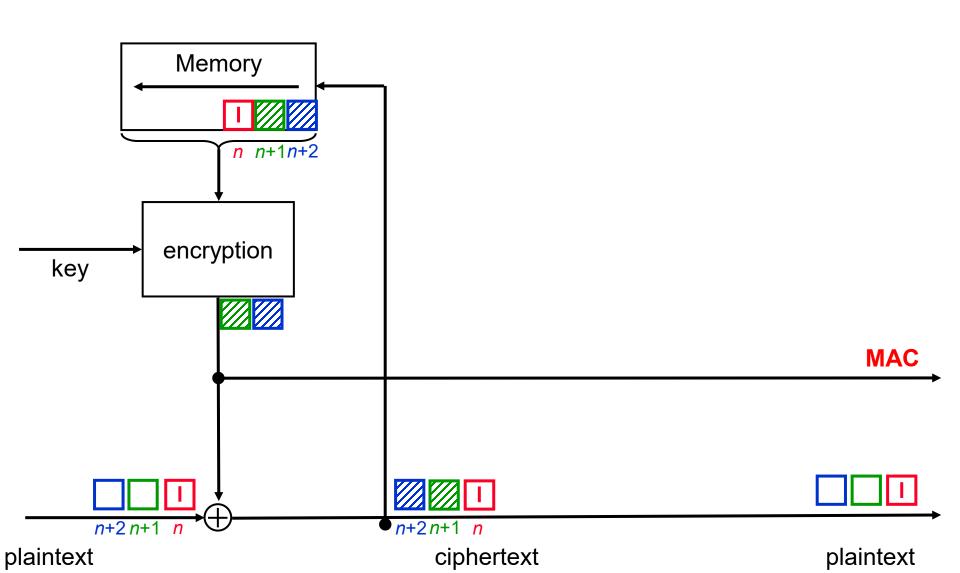
Cipher FeedBack (CFB) (2)

- *b* Block length
- *a* Length of the output unit, $a \le b$
- *r* Length of the feedback unit, $r \le b$
- ⊕ Addition mod appropriately chosen modulus
- ⊖ Subtraction mod appropriately chosen modulus



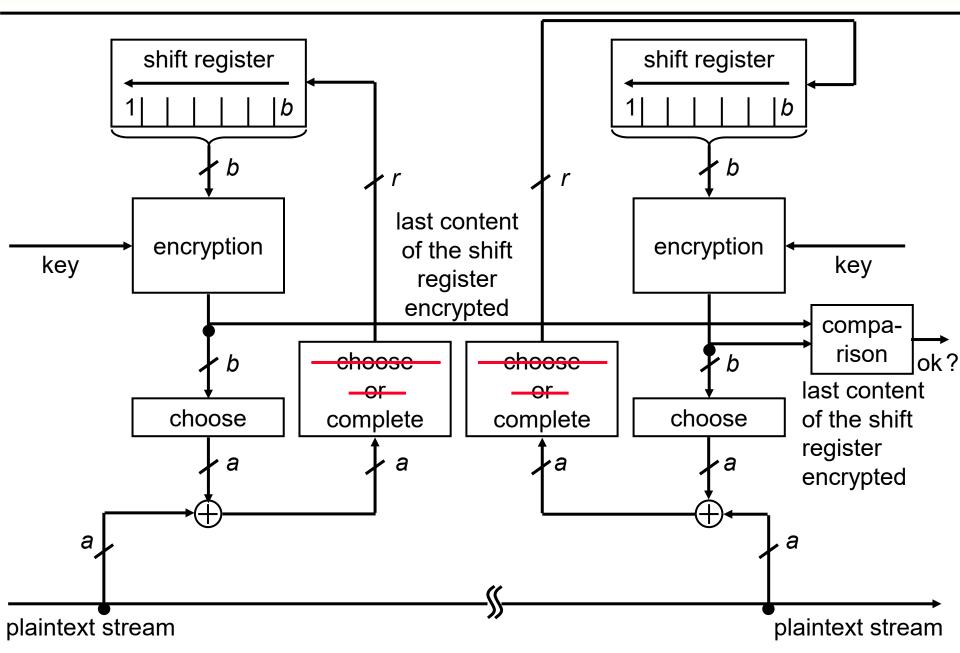








CFB for authentication





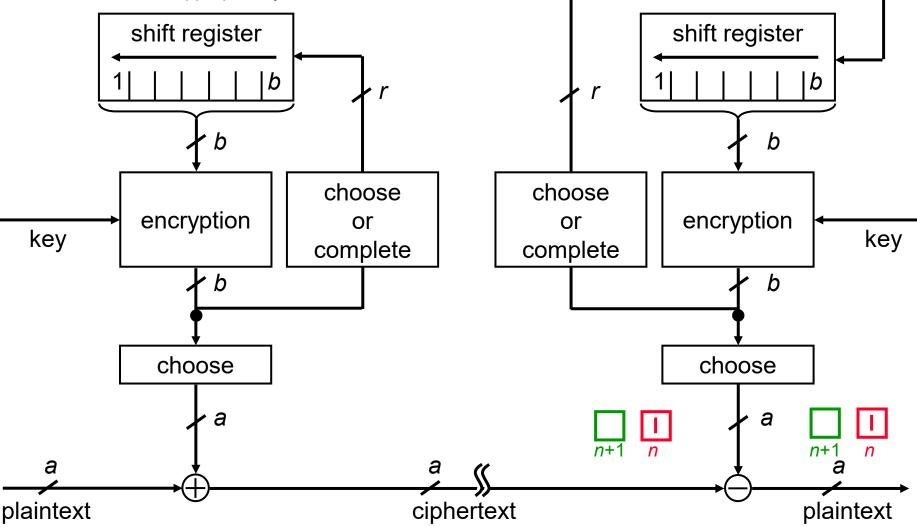
Output FeedBack (OFB)

symmetric;

synchronous

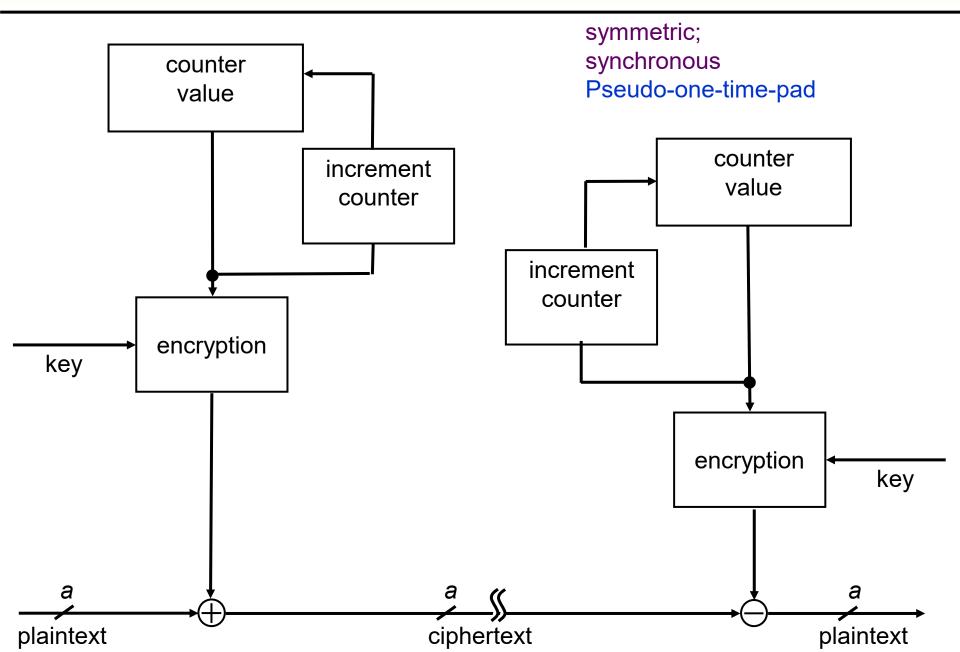
Pseudo-one-time-pad

- b Block length
- *a* Length of the output unit, $a \le b$
- *r* Length of the feedback unit, $r \le b$
- ⊕ Addition mod appropriately chosen modulus
- Θ Subtraction mod appropriately chosen modulus

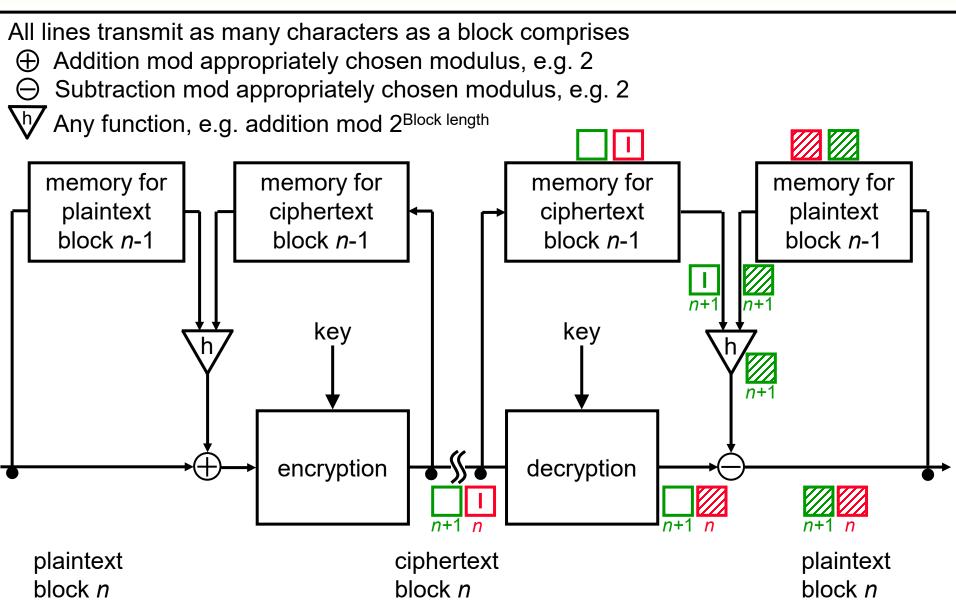




Counter Mode (CTR)

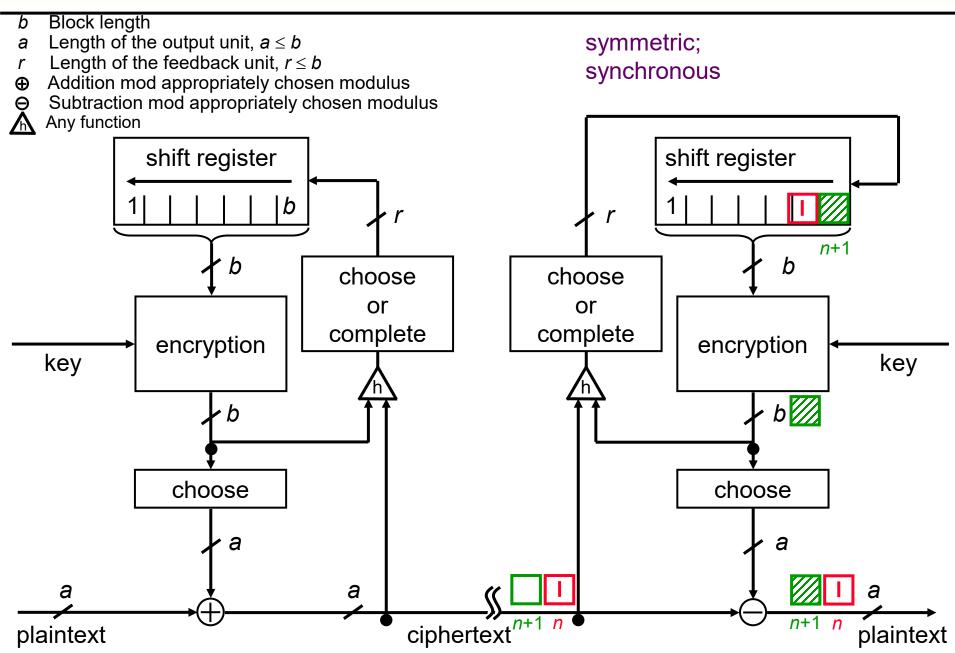






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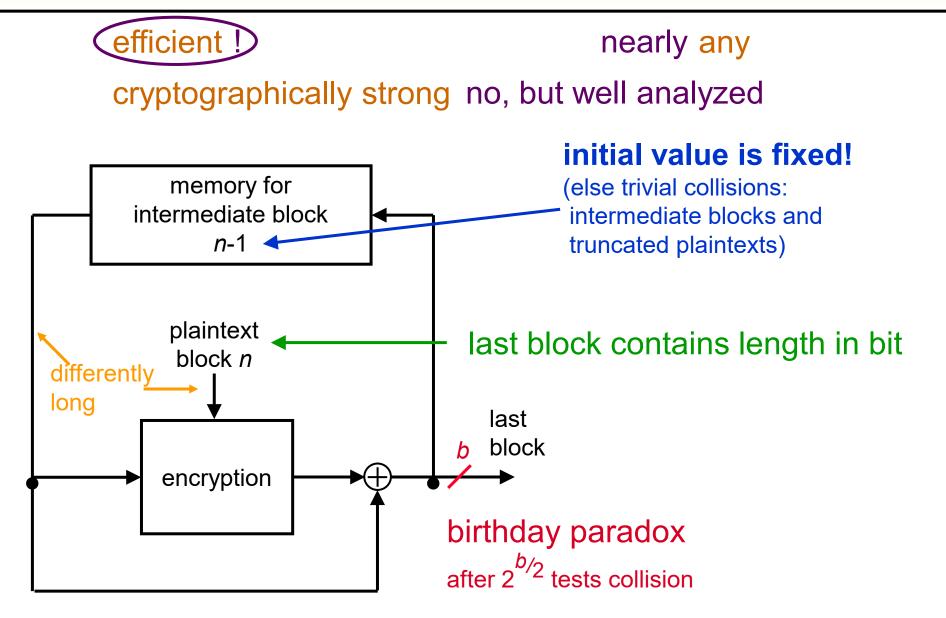






	ECB	CBC	PCBC	CFB	OFB	OCFB
Utilization of indeterministic block cipher	+ possible			- impossible		
Use of an asymmetric block cipher results in	+ asymmetric stream cipher			- symmetric stream cipher		
Length of the units of encryption	- determined by block length of the block cipher			+ user-defined		
Error extension	only within the block (assuming the borders of blocks are preserved)	2 blocks (assuming the borders of blocks are preserved)	potentially unlimited	1 + [<i>b</i> / <i>r</i>] blocks, if error placed rightmost, else possibly one block less	none as long as no bits are lost or added	potentially unlimited
Qualified also for authentication?	yes, if redundancy within every block	yes, if deterministic block cipher	yes, even concealment in the same pass	yes, if deterministic block cipher	yes, if adequate redundancy	yes, even concealment in the same pass

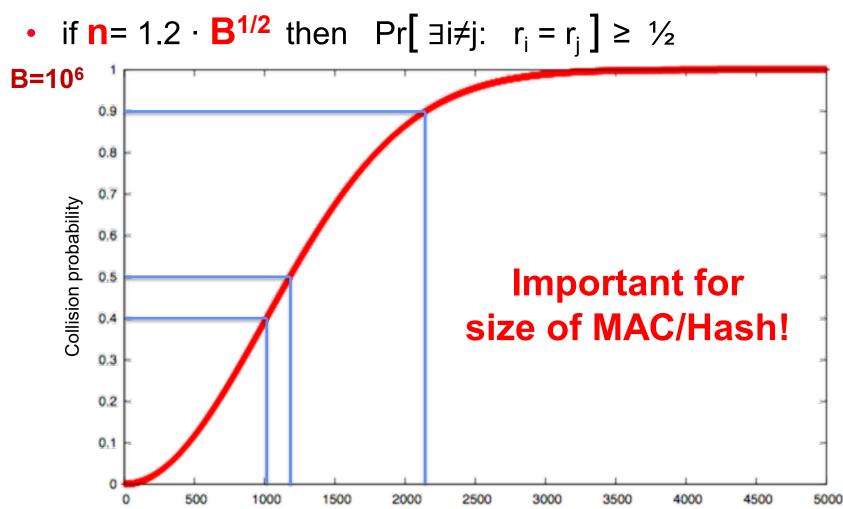
Collision-resistant hash function using determ. block cipher



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The Birthday Paradox

Let r₁, ..., r_n ∈ {1,...,B} be random integers, chosen independent and identically distributed (iid).

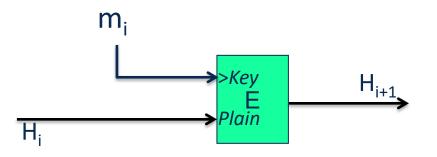


[#] samples n



E: $K \times \{0,1\}^n \longrightarrow \{0,1\}^n$ a block cipher.

Construct cascade, for compression encrypt message blocks:



What's wrong with that?

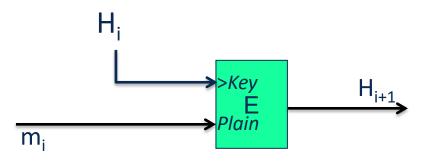
 $H_{i+1} = E(m_i, H_i)$

Can you find a collision on this compression function? $H_{i+1} = E(m', D(m', H_{i+1}))$



E: $K \times \{0,1\}^n \longrightarrow \{0,1\}^n$ a block cipher.

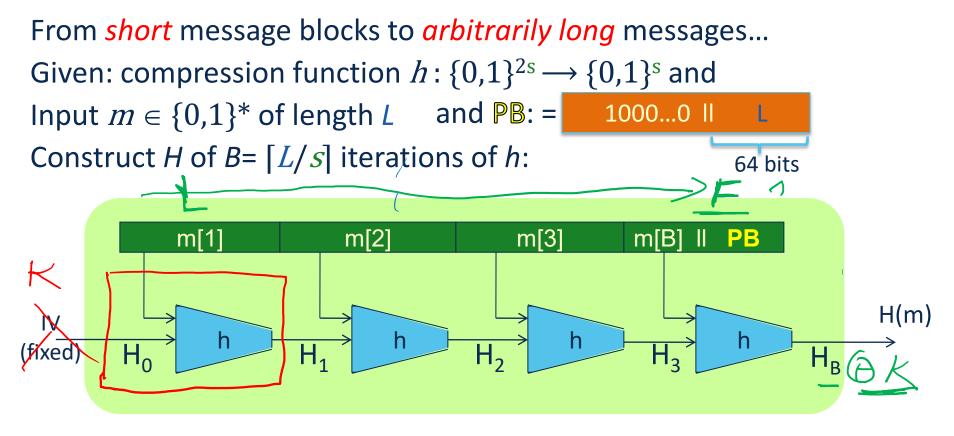
Construct cascade, for compression encrypt message blocks:



Message $m=m_1 | m_2 \rightarrow H_0 = IV \rightarrow H_1 = E(IV, m_1) \rightarrow H_2 = E(H_1, m_2) = h$

Manipulated Message: $m'=m'_1 | m'_2 = m'_1 | D(E(IV, m'_1), H_2)$ $\Rightarrow H_0 = IV \Rightarrow H_1 = E(IV, m'_1) \Rightarrow H_2 = E(H_1, m'_2) = E(H_1, D(E(IV, m'_1), H_2)) = E(H_1, D(E(IV, m'_1), H_2)) = H_2 = h$

TECHNISCHE UNIVERSITAT The Merkle-Damgård construction



If h is a fixed length CRHF, then H_{is} an arbitrary length CRHF **Proof**: either m=m', or $H_{B-i}(m[B-i])=H_{B-i}(m'[B-i])$ no collision $MA \subset m'$ collision on h Val i d

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Privacy and Security



practically important:patent exhausted before that of RSA \rightarrow used in PGP from Version 5 ontheoretically important:steganography using public keys

based on difficulty to calculate **discrete logarithms**

Given a prime number p and g a generator of Z_{p}^{*}

x is the **discrete logarithm** of **h** to basis **g** modulo **p**:

 $\mathbf{x} = \log_{\mathbf{g}}(\mathbf{h}) \mod \mathbf{p}$

discrete logarithm assumption



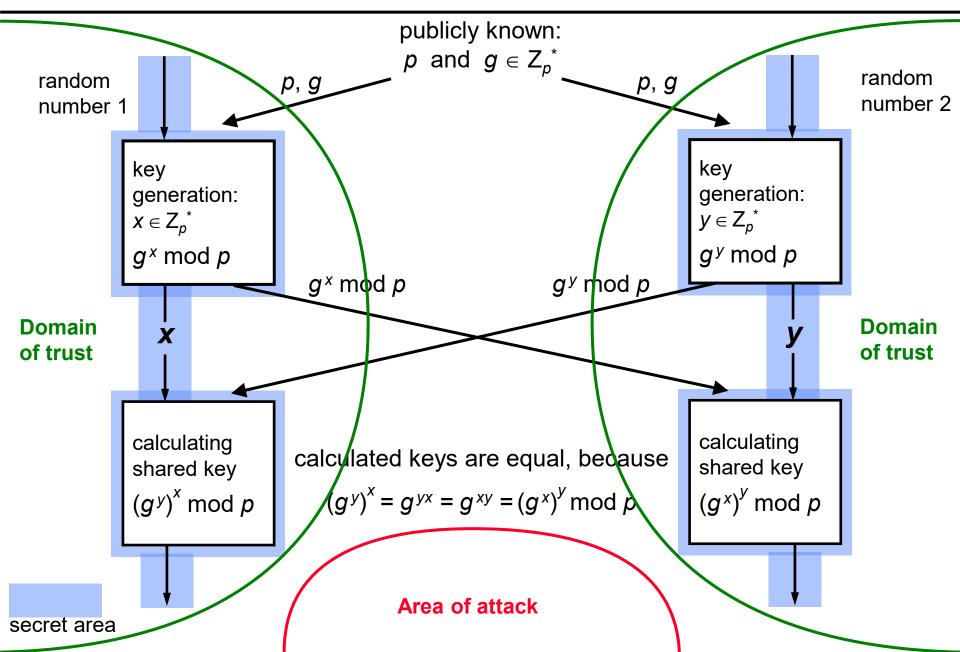
 $\forall \mathsf{PPA} \ \mathcal{DL}$ (probabilistic polynomial algorithm, which tries to calculate discrete logarithms) \forall polynomials Q $\exists L \forall \mathcal{L} \geq L$: (asymptotically holds) If p is a random prime of length \mathcal{U} thereafter g is chosen randomly within the generators of Z_{ρ}^{*} x is chosen randomly in Z_{ρ}^{*} and $g^x = h \mod p$ $\mathcal{W}(\mathcal{D}\mathcal{L}(p,g,h)=x) \leq \frac{1}{Q(\Delta)}$ (probability that \mathcal{DL} really calculates the discrete logarithm, decreases faster than $\frac{1}{any polynomial}$)

trustworthy ??

practically as well analyzed as the assumption factoring is hard



Diffie-Hellman key agreement (2)



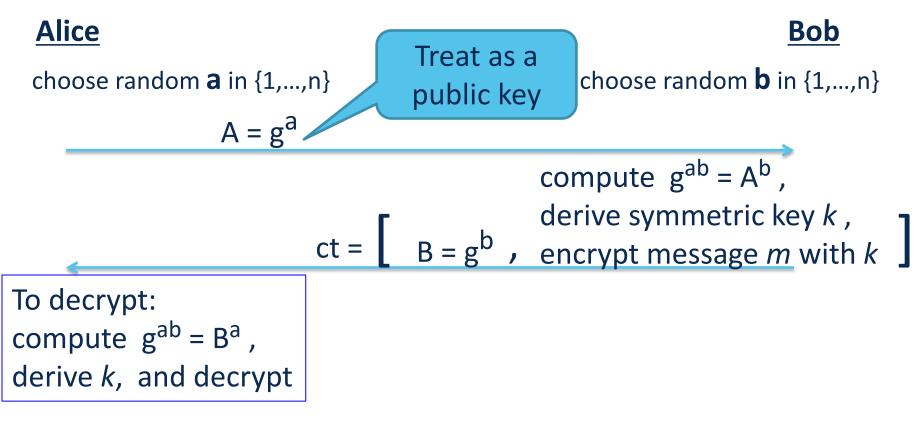


Diffie-Hellman (DH) assumption: Given p, g, $g^x \mod p$ and $g^y \mod p$ Calculating $g^{xy} \mod p$ is difficult.

DH assumption is stronger than the discrete logarithm assumption

- Able to calculate discrete Logs ⇒ DH is broken.
 Calculate from *p*, *g*, *g^x* mod *p* and *g^y* mod *p* either
 x or *y*. Calculate *g^{xy}* mod *p* as the corresponding partner of the DH key agreement.
- Until now it couldn't be shown:
 Using p, g, g^x mod p, g^y mod p and g^{xy} mod p
 either x or y can be calculated.

Fix a finite cyclic group G (e.g $G = (Z_p)^*$) of order *n* Fix a generator *g* in G (i.e. $G = \{1, g, g^2, g^3, ..., g^{n-1}\}$)





- G: finite cyclic group of order *n*
- (E_s, D_s) : symmetric auth. encryption defined over (K,M,C)
- $H: G \times G \longrightarrow K$ a hash function

Construct a pub-key encryption system (Gen, E, D):

Key generation Gen:

- choose random generator g in G and random a in Z_n
- output sk = a , pk = (g, h=g^a)

 $\begin{array}{l} \underline{\mathsf{E(pk=(g,h), m)}:}\\ b \leftarrow Z_n, \ u \leftarrow g^b, \ v \leftarrow h^b = g^{a \cdot b}\\ k \leftarrow H(u,v), \ c \leftarrow E_s(k,m)\\ output \ (u,c) \end{array}$

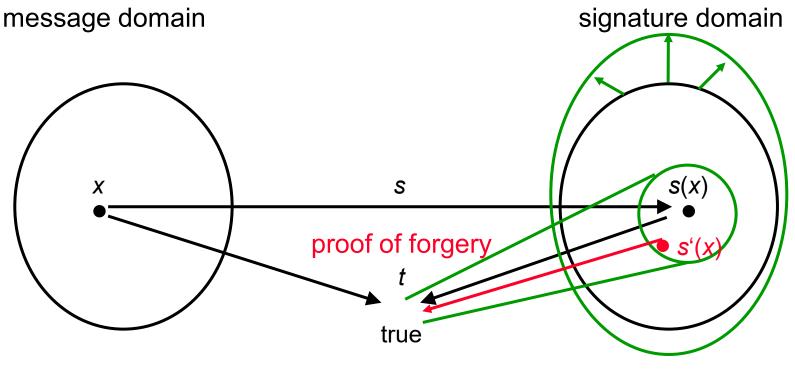
 $\begin{array}{l} \underline{\mathsf{D(sk=a,(u,c))}:}\\ \mathbf{v} \leftarrow \mathbf{u}^{a} = \mathbf{g}^{b \cdot a}\\ k \leftarrow \mathsf{H}(u,v), \quad \mathbf{m} \leftarrow \mathsf{D}_{s}(k,c)\\ \text{output} \quad \mathbf{m} \end{array}$



Security is asymmetric, too

usually: unconditionally secure for recipient only cryptographically secure for signer

new: signer is absolutely secure against breaking his signatures provable only cryptographically secure for recipient

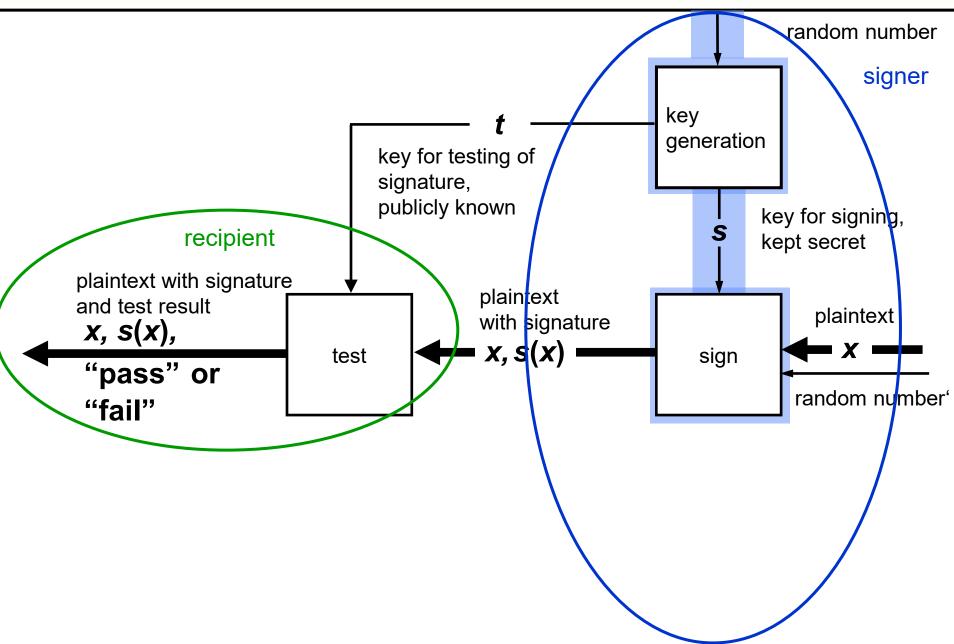


distribution of risks if signature is forged: 1. recipient

- 2. insurance or system operator
- 3. signer

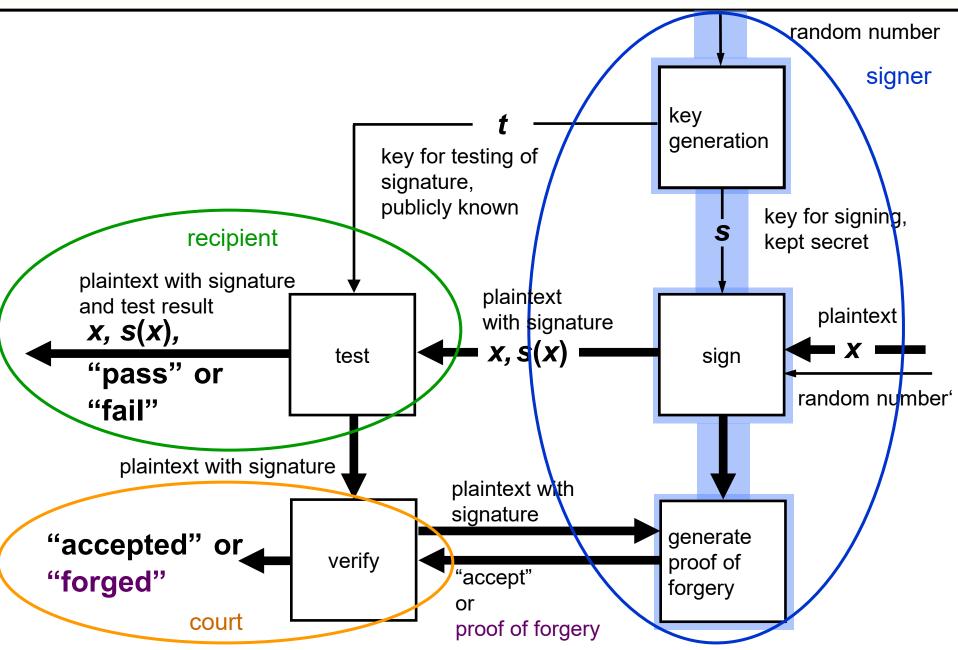
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Fail-stop signature system

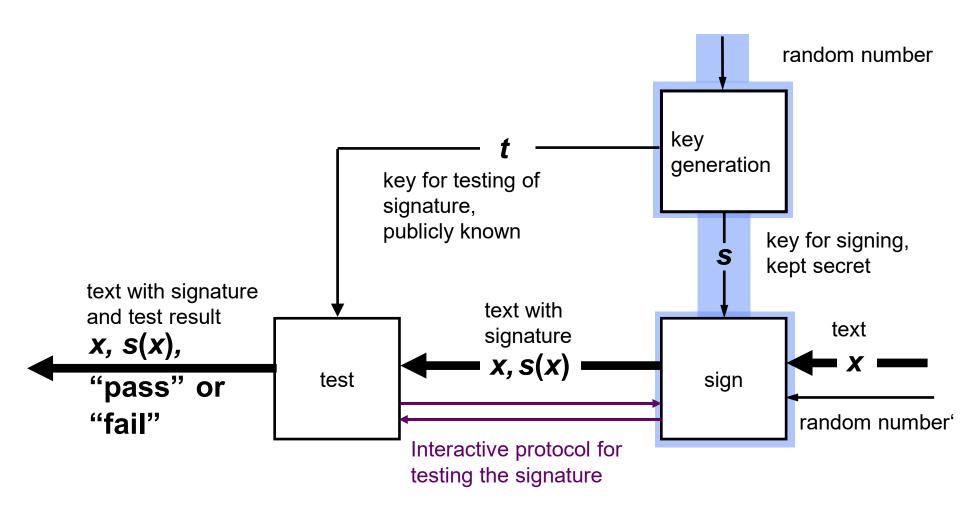




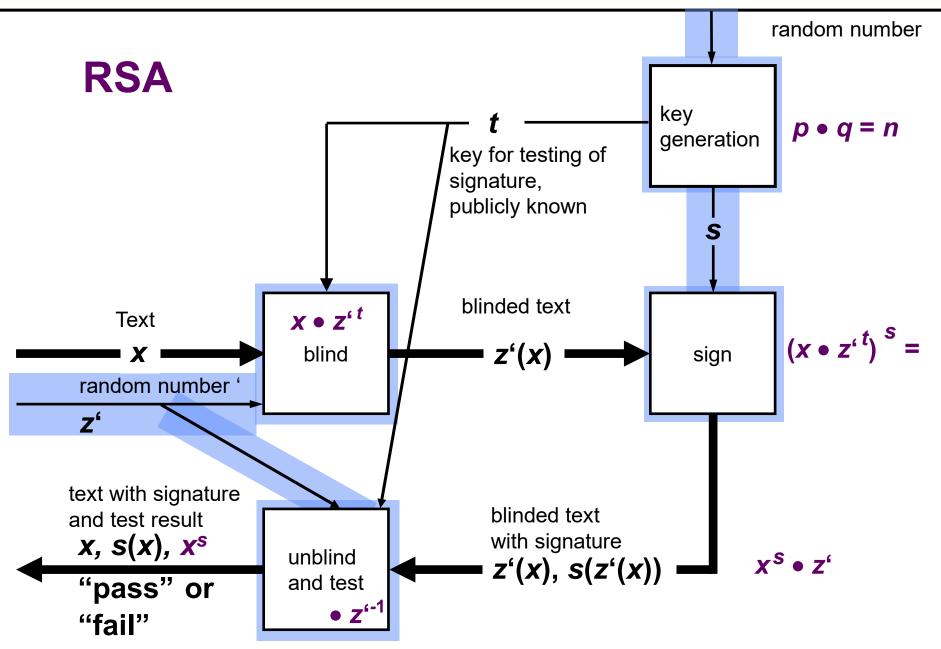
Fail-stop signature system







Signature system for blindly providing of signatures





Threshold scheme:

Secret S

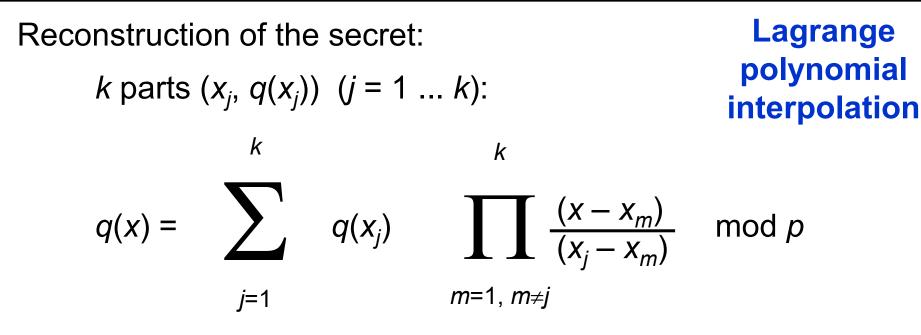
- *n* parts
- k parts: efficient reconstruction of S
- k-1 parts: no information about S

Implementation: polynomial interpolation (Shamir, 1979)

Decomposition of the secret:

Let secret *S* be an element of Z_p , *p* being a prime number. Polynomial q(x) of degree *k*-1: Choose $a_1, a_2, ..., a_{k-1}$ randomly in Z_p $q(x) := S + a_1x + a_2x^2 + ... + a_{k-1}x^{k-1}$ *n* parts (i, q(i)) with $1 \le i \le n$, where n < p.





The secret S is q(0).

Sketch of proof:

- 1. *k*-1 parts (*j*, *q*(*j*)) deliver no information about *S*, because for each value of *S* there is still exactly one polynomial of degree *k*-1.
- 2. correct degree *k*-1; delivers for any argument x_j the value $q(x_j)$ (because product delivers on insertion of x_j for *x* the value 1 and on insertion of all other x_j for *x* the value 0).



Polynomial interpolation is Homomorphism w.r.t. addition

Addition of the parts \Rightarrow Addition of the secrets

Share refreshing

- 1.) Choose random polynomial q' for S' = 0
- 2.) Distribute the *n* parts $(i, q^{(i)})$
- 3.) Everyone adds his "new" part to his "old" part
 - \rightarrow "new" random polynomial q+q with "old" secret S
- Repeat this, so that anyone chooses the random polynomial once
- Use *verifiable secret sharing*, so that anyone can test that polynomials are generated correctly.