Secure Computation

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Some slides taken from lectures of Ran Cohen, Yehuda Lindell, Mike Rosulek

Definition:

- Secure computation (SC) (also known as Secure multi-party computation (SMPC), multi-party computation (MPC) is a subfield of cryptography with the goal of creating methods for parties to jointly compute a function over their inputs while keeping those inputs private.

- SC protocols can enable data scientists and analysts to compliantly, securely, and privately compute on distributed data without ever exposing or moving it.
- Researchers are making SC faster and easier to use for application software developers

Scenario: Private Auction

Many parties wish to execute a private auction

- The highest bid wins
- Only the highest bid (and bidder) is revealed

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Solution: use a trusted auctioneer

Secure Computation

- In the scenario the solution of an external trusted third party works
- Trusting a third party is a very strong assumption
- Can we do better?
- We would like a solution with the same security guarantees, but **without** using any trusted party

Secure Computation

Goal: use a protocol to emulate the trusted party

The setting

- Parties $P_1, \ldots, P_n$
- Party P_i has private input x_i
- The parties wish to jointly compute a function $y = f(x_1, \ldots, x_n)$
- The computation must preserve certain security properties, even is some of the parties collude and maliciously attack the protocol
- Normally, this is modeled by an external adversary $\mathcal{A}$ that corrupts some parties and coordinates their actions

Security Requirements

- **Correctness**: parties obtain correct output (even if some parties misbehave)
- **Privacy**: only the output is learned (nothing else)
- **Independence of inputs**: parties cannot choose their inputs as a function of other parties' inputs
- **Fairness**: if one party learns the output, then all parties learn the output
- **Guaranteed output delivery**: all honest parties learn the output

Auction Example – Security Requirements

- **Correctness**: $\mathcal{A}$ can't win using lower bid than the highest
- **Privacy**: $\mathcal{A}$ learns an upper bound on all inputs, nothing else
- **Independence of inputs**: $\mathcal{A}$ can't bid one dollar more than the highest (honest) bid
- **Fairness**: $\mathcal{A}$ can't abort the auction if his bid isn't the highest (i.e., after learning the result)
- **Guaranteed output delivery**: $\mathcal{A}$ can't abort (stronger than fairness, no DoS attacks)

Who is Richer?

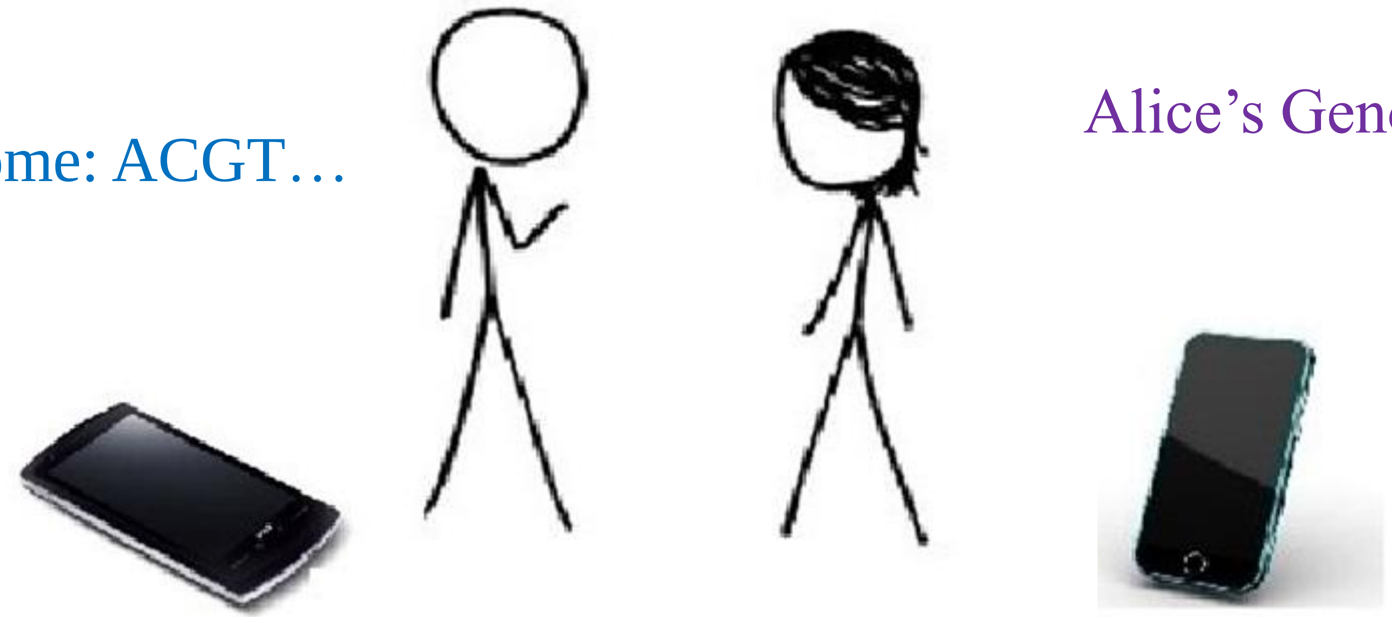
Millionaires’ Problem

X > Y ?!!

Secure string matching

Bob’s Genome: ACGT…

Alice’s Genome: ACTG…



Can Alice and Bob compute a function of their private data without exposing anything about their data besides the result?

Secret Sharing

s from F_p

$F_p = (Z_p, +, \bullet)$ is a field

>> Choose random shares $s_1,..s_n$ from F_p s. t. $s_1 + \ldots + s_n = s$

>> $S = \{s_1,.. s_n\}$

$S \backslash s_1$  $S \backslash s_2$  .... $S \backslash s_n$ 

>> Together all the parties know S

>> Individual party has no information about S.

Secure Addition $y = x_1 + x_2 + x_3$ (assume n=3 parties)

No party even with unbounded power learns nothing more than y !

x_1 $\quad$ x_2 $\quad$ x_3

$x_{11}\ x_{12}\ x_{13}\quad x_{21}\ x_{22}\ x_{23}\quad x_{31}\ x_{32}\ x_{33}$

$y = s_1 + s_2 + s_3$

P_1  $x_{11} + x_{21} + x_{31} = s_1$

P_2  $x_{12} + x_{22} + x_{321} = s_2$

P_3  $x_{13} + x_{23} + x_{33} = s_3$

$s_1, s_2, s_3 \rightarrow P_i$

Secure bit multiplication $y = x_1 \bullet x_2$

$$y = x_1 \bullet x_2$$
$$= (x_{11} + x_{12}) \bullet (x_{21} + x_{22})$$
$$= (\underline{x_{11} \bullet x_{21}} + x_{11} \bullet x_{22} + x_{12} \bullet x_{21} + \underline{x_{12} \bullet x_{22}})$$

x_1 x_2

x_{11} x_{12} x_{21} x_{22}

$x_{12} \bullet x_{22} = x_{12} \bullet x_{22}$

$x_{11} \bullet x_{21} = x_{11} \bullet x_{21}$

Oblivious Transfer (OT)

x_0

x_1

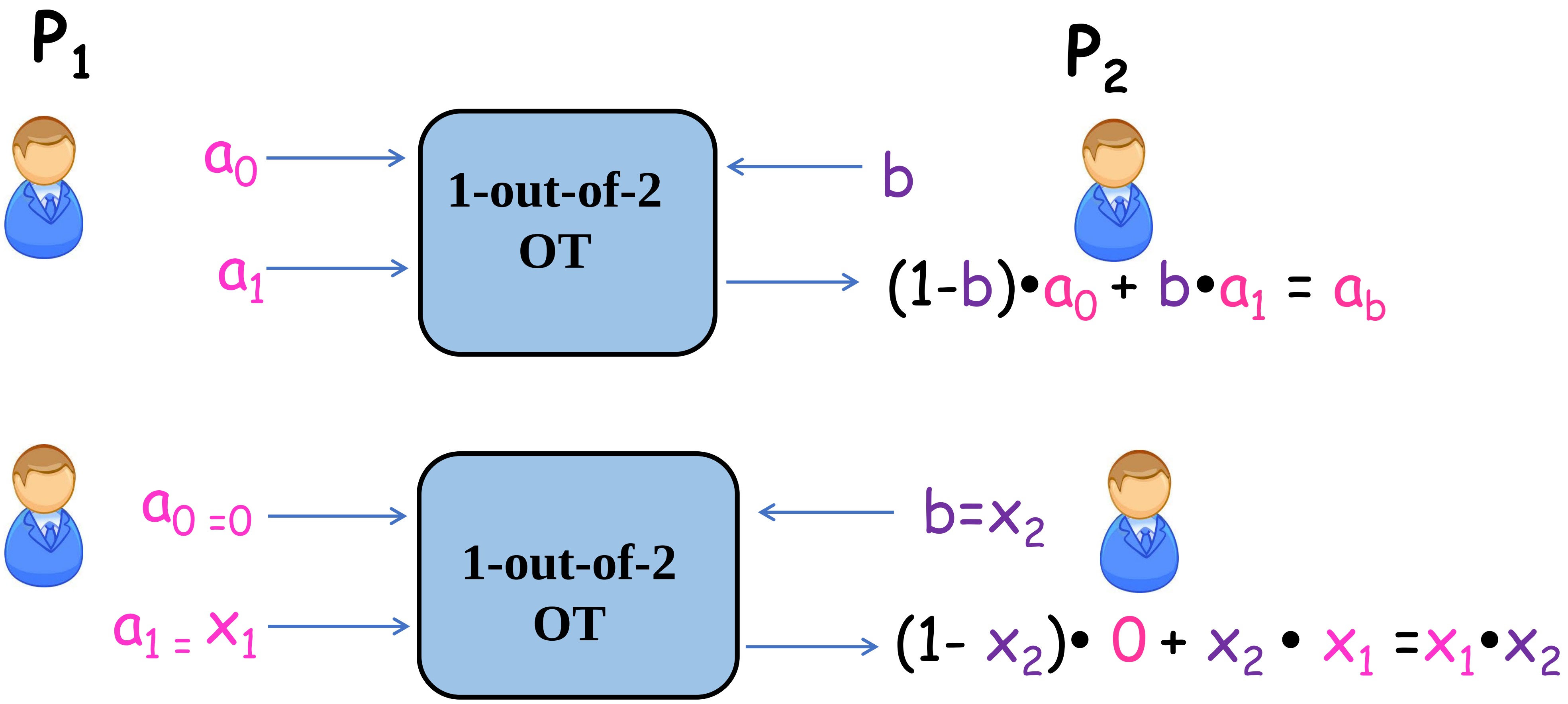
Choice bit: $\boldsymbol{b}$

Sender

Receiver

- Sender holds two bits x_0 and x_1.
- Receiver holds a choice bit b.
- Receiver should learn x_b, sender should learn nothing.

Secure bit multiplication $y = x_1 \bullet x_2$



How to Define Security

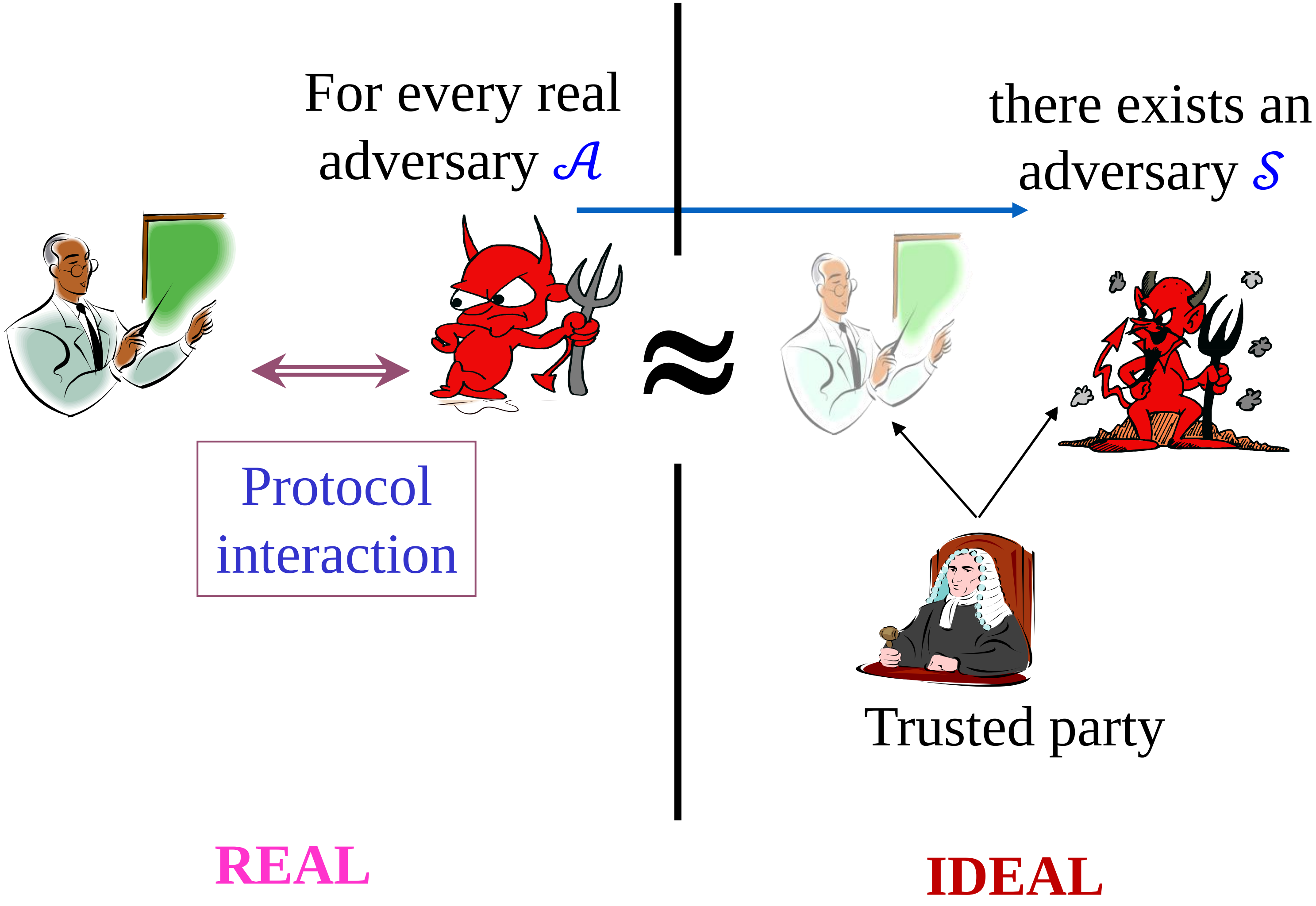
Option 1: property-based definition

- Define a list of security requirements for the task
- Analyze security concerns for each specific problem
- Difficult to analyze complex tasks
- How do we know if all concerns are covered?
- Definitions are application dependent (no general results, need to redefine each time).

- **Option 2:** the real/ideal paradigm
- Whatever an adversary can achieve by attacking a real protocol can also be achieved by attacking an ideal computation involving a trusted party
- Formalized via a simulator
- The real/ideal model paradigm:
  - Ideal model: parties send inputs to a trusted party, who computes the function and sends the outputs.
  - Real model: parties run a real protocol with no trusted help.

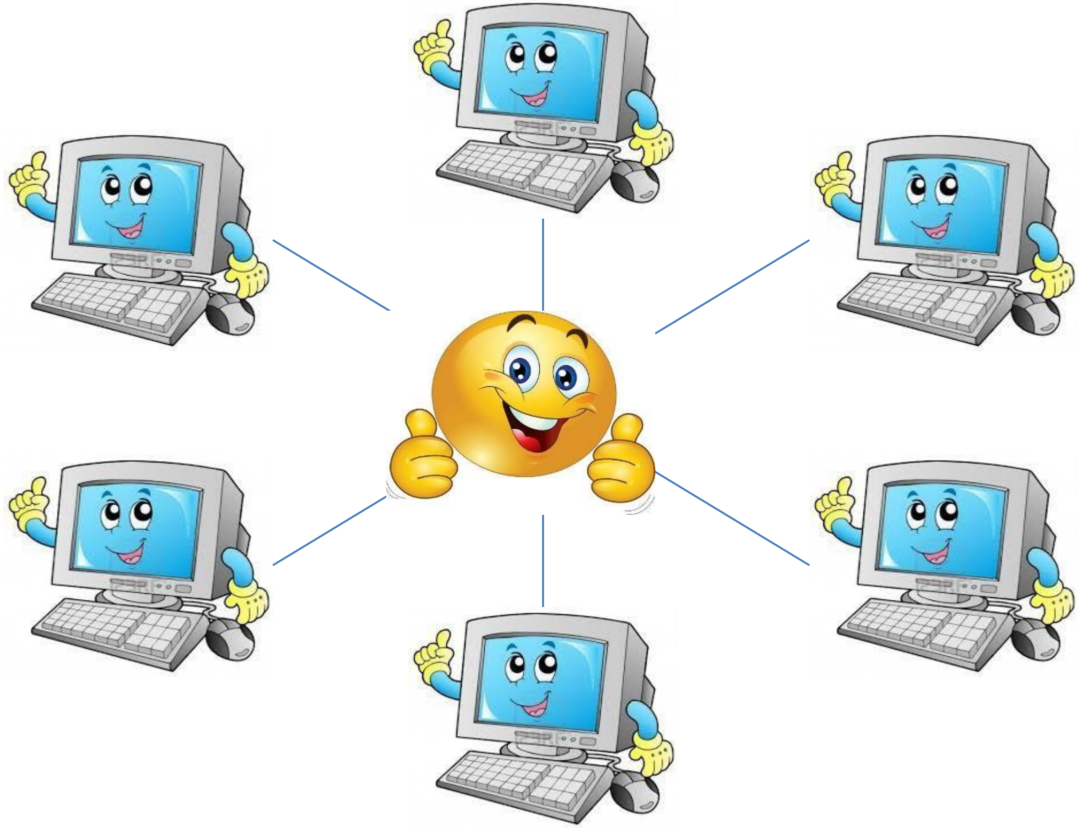
- Informally: a protocol is secure if any attack on a real protocol can be carried out in the ideal model.
- Since **no** attacks can be carried out in the ideal model, security is implied.

The Security Definition:



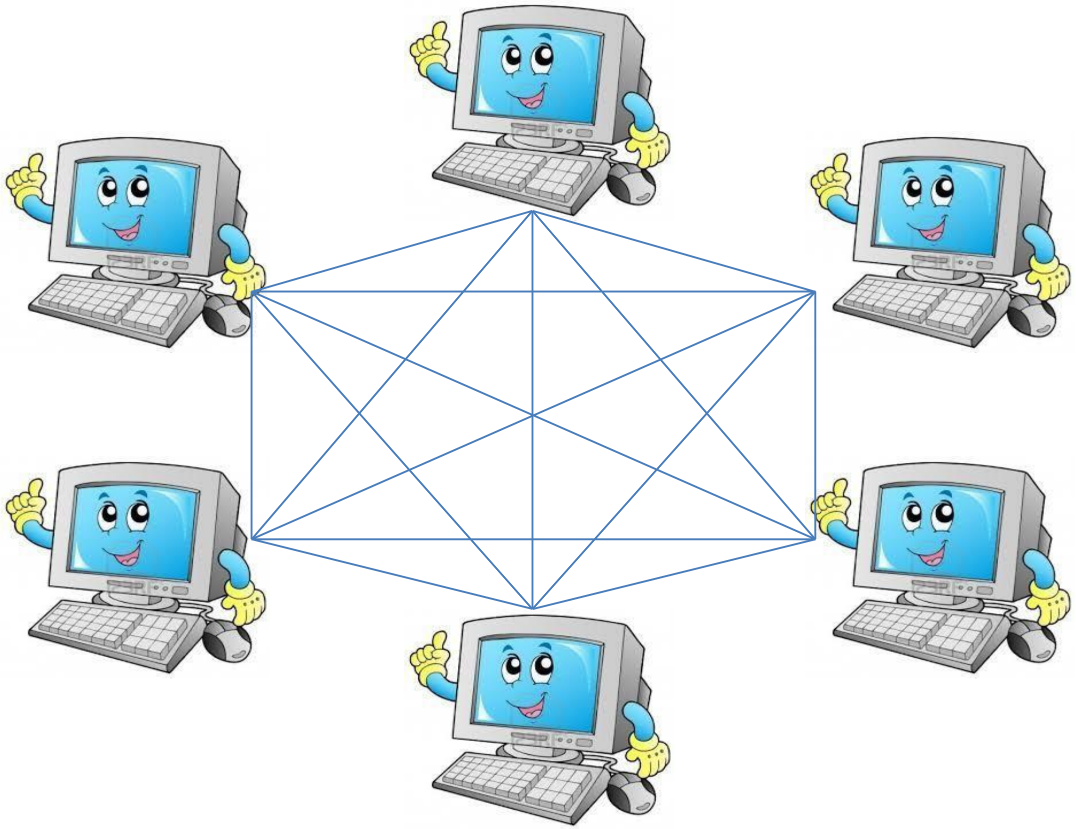
Ideal World

1) Each party sends its input to the trusted party
2) The trusted party computes $y = f(x_1, \ldots, x_n)$
3) Trusted party sends y to each party

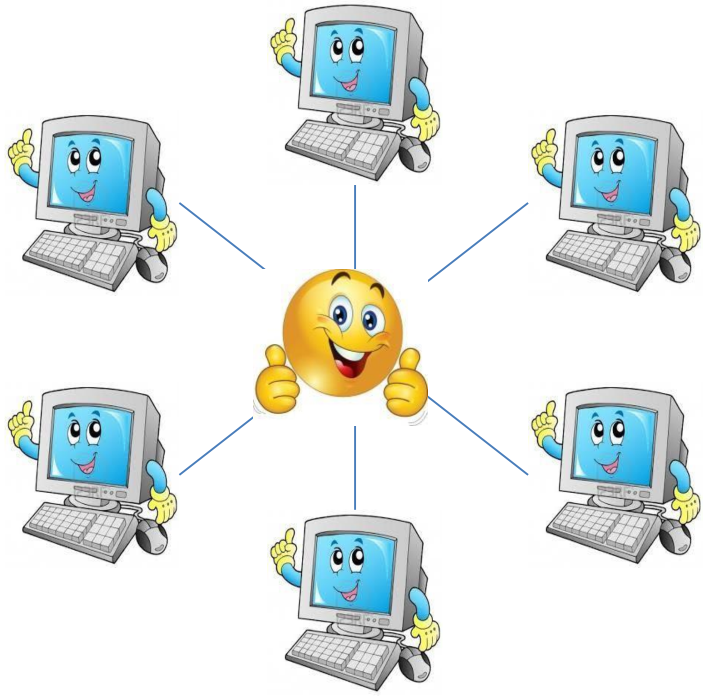


Real World

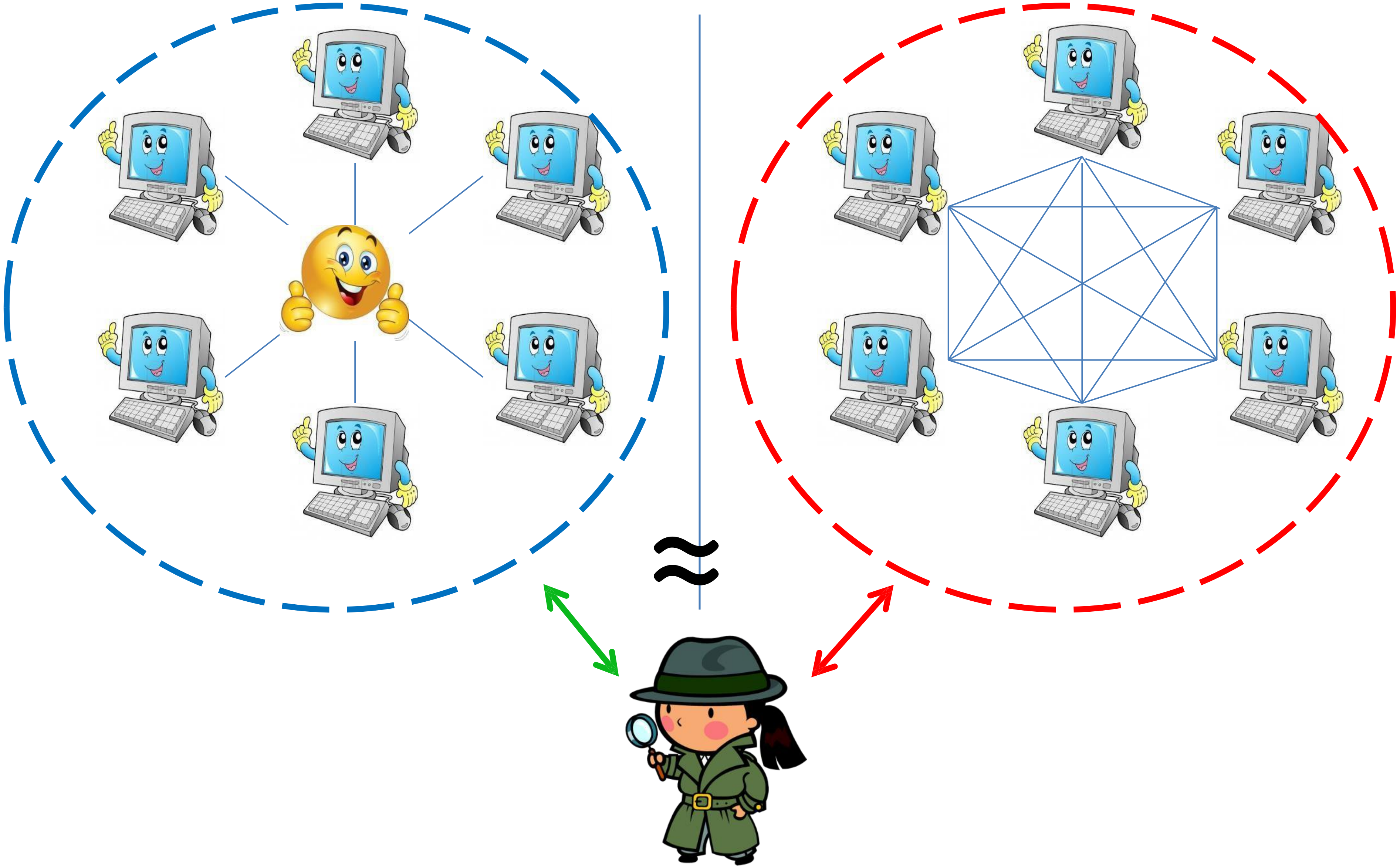
Parties run a protocol π on inputs $(x_1, \ldots, x_n)$



Simulation-Based Security

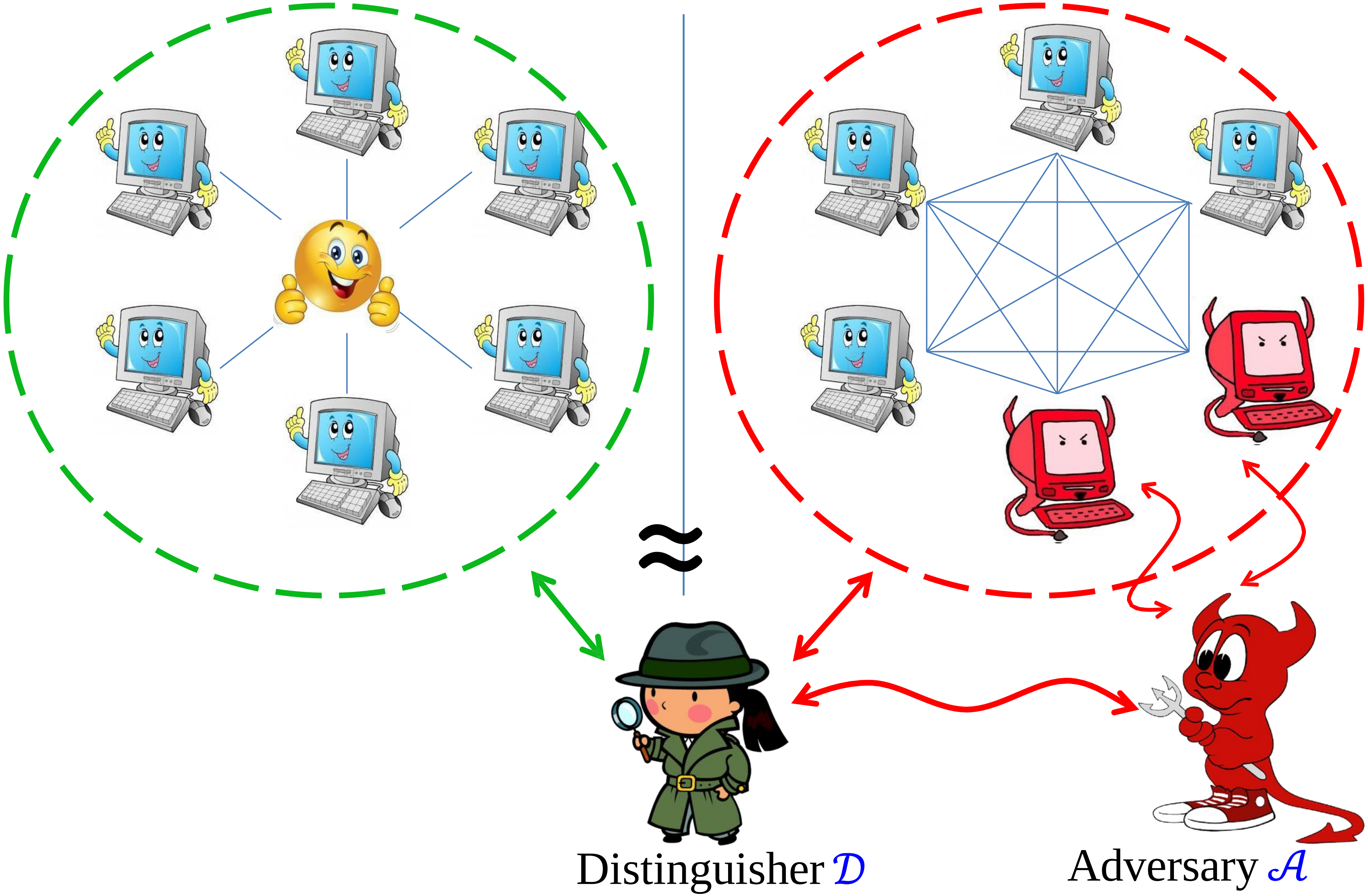


Simulation-Based Security



Distinguisher $\mathcal{D}$

Simulation-Based Security



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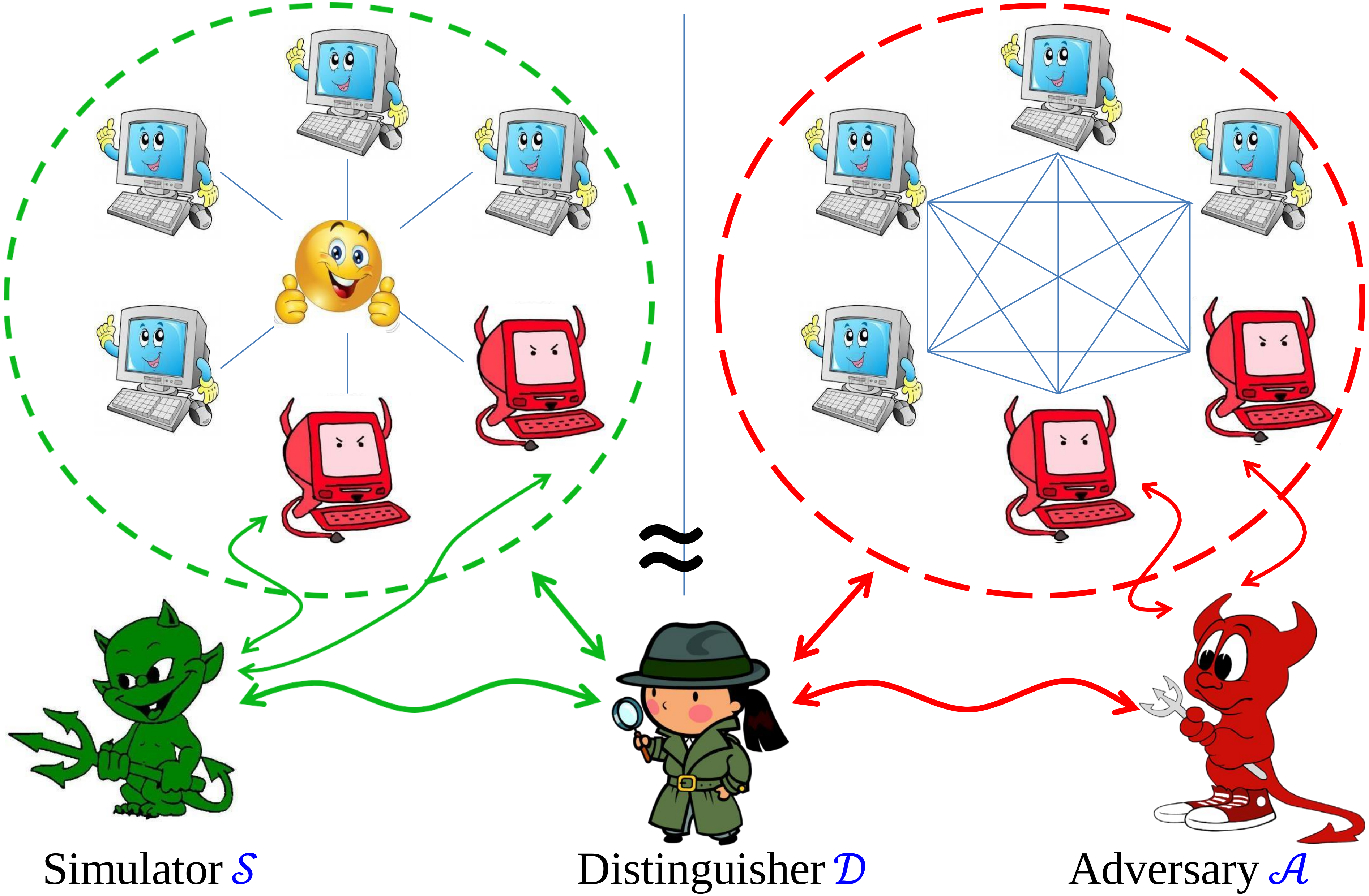
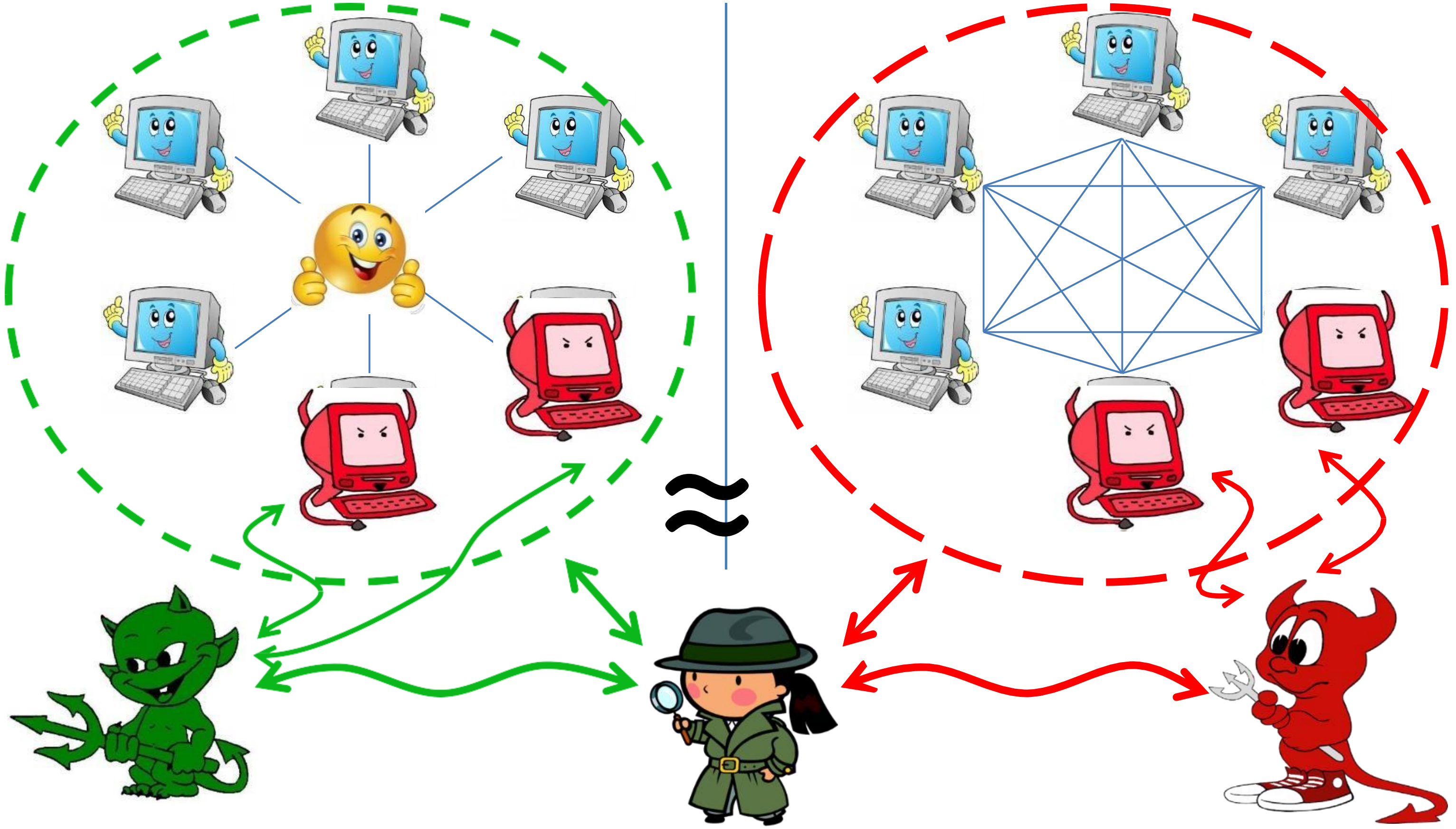


Simulation-Based Security

The distinguisher $\mathcal{D}$:

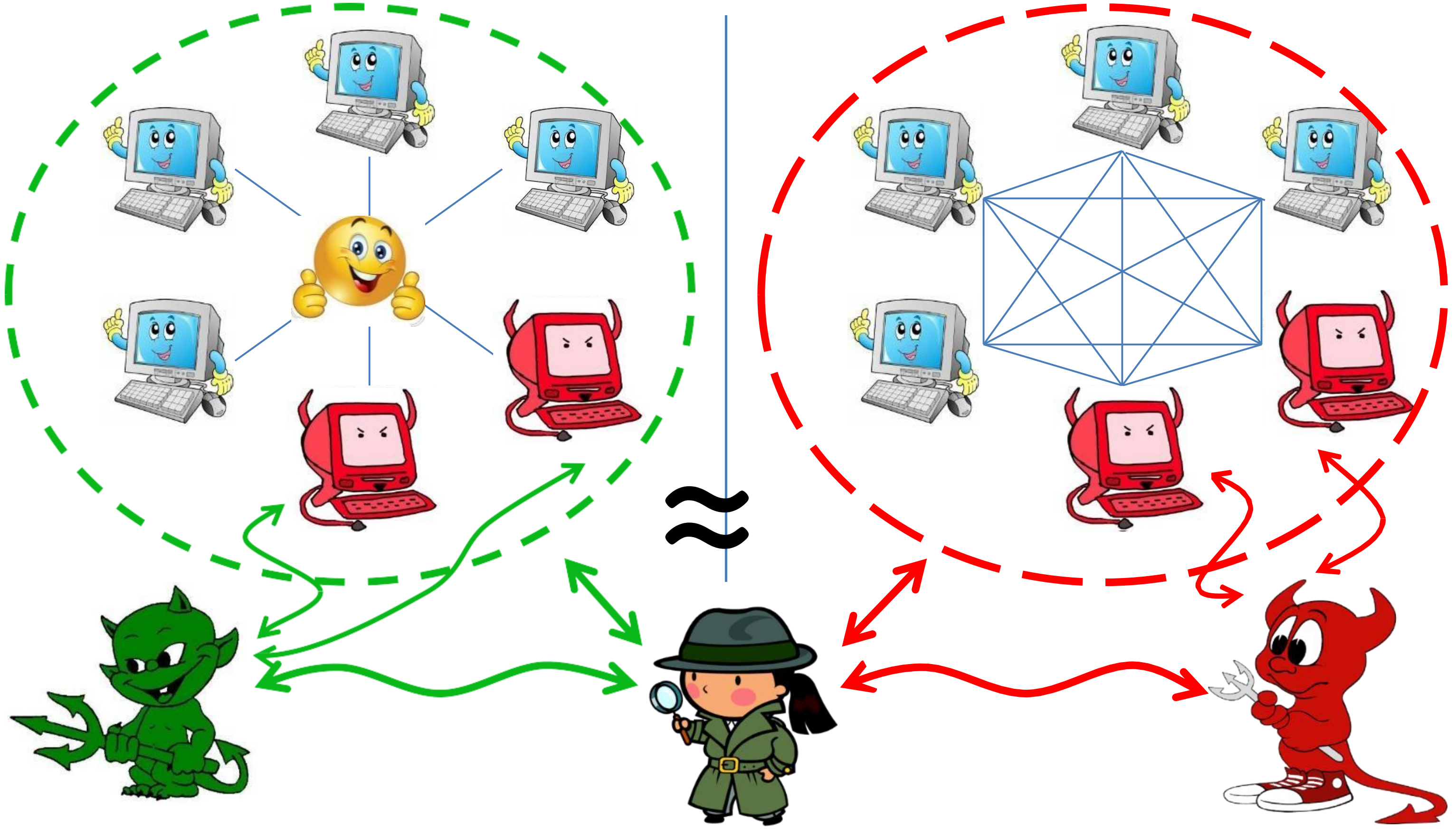
- Gives inputs to parties
- Gets back output from parties and from adversary/simulator
- Guesses which world it is real/ideal

Protocol π securely computes f if $\forall \mathcal{A}\ \exists \mathcal{S}\ \forall \mathcal{D}$ distinguishing success is “small”



Sanity check

- ✓ Correctness
- ✓ Privacy
- ✓ Independence of inputs
- ✓ Fairness
- ✓ Guaranteed output delivery



The Definition Cont'd

A definition of an SC task involves defining:

- **Functionality**: what do we want to compute?
- **Security type**: how strong protection do we want?
- **Adversarial model**: what do we want to protect against?
- **Network model**: in what setting are we going to do it?

The Functionality

- The code of the trusted party
- Captures inevitable vulnerabilities
- Sometimes useful to let the functionality talk to the ideal-world adversary (simulator)
- We will focus on secure function evaluation (SFE), the trusted party computes $y = f(x_1, \ldots, x_n)$

Security Type

- **Computational**: a probabilistic polynomial time (PPT) distinguisher
  - The real & ideal worlds are computationally indistinguishable
- **Statistical**: all-powerful distinguisher, negligible error probability
  - The real & ideal worlds are statistically close
- **Perfect**: all-powerful distinguisher, zero error probability
  - The real & ideal worlds are identically distributed

Adversarial Model

- **Adversarial behavior**
  - **Semi honest**: honest-but-curious. corrupted parties follow the protocol honestly, $\mathcal{A}$ tries to learn more information.
  - **Malicious**: corrupted parties can deviate from the protocol in an arbitrary way

- **Adversarial power**
  - **Polynomial time**: the adversary is allowed to run in (probabilistic) polynomial time (and sometimes, expected polynomial time), computational security
  - **Computationally unbounded**: the adversary has no computational limits whatsoever, information-theoretic security

Adversarial Model

- **Adversarial corruption**
  - **Static**: the set of corrupted parties is defined before the execution of the protocol begins. Honest parties are always honest, corrupted parties are always corrupted
  - **Adaptive**: $\mathcal{A}$ can decide which parties to corrupt during the course of the protocol, based on information it dynamically learns
  - **Mobile**: $\mathcal{A}$ can jump between parties. Honest parties can become corrupted, corrupted parties can become honest again

Communication Model

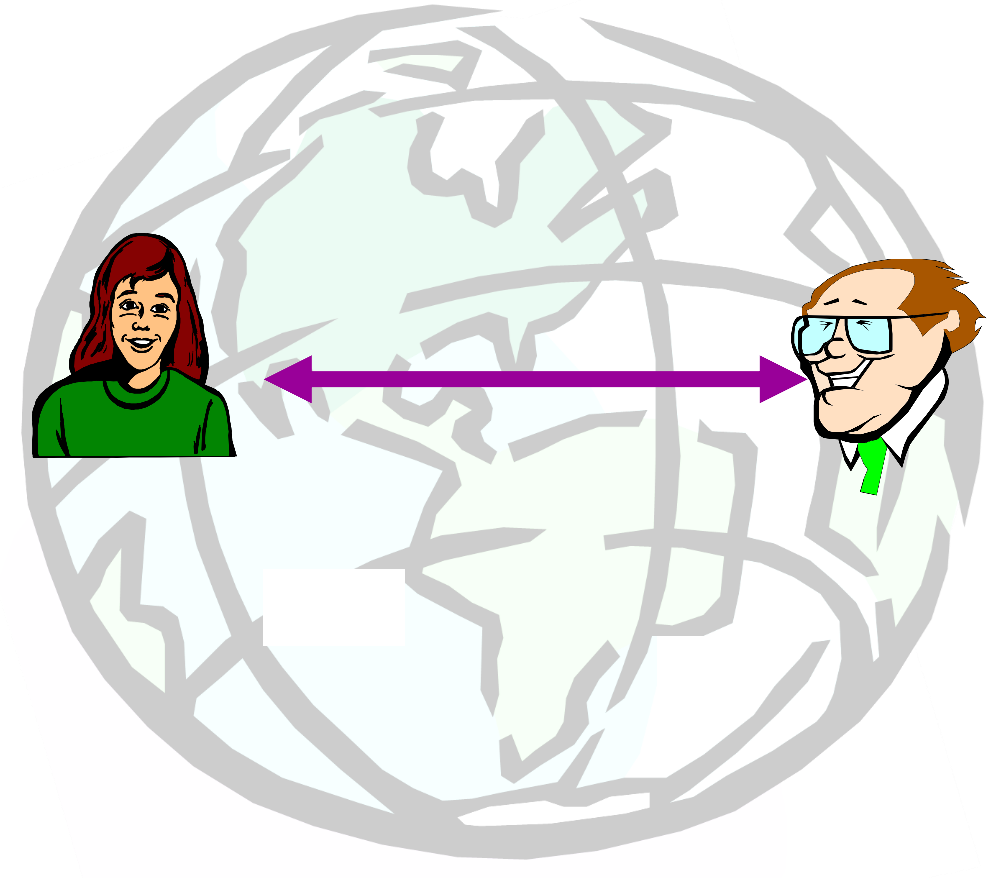
- **Point-to-point:** fully connected network of pairwise channels.
- **Broadcast:** additional broadcast channel

- **Message delivery:**
  - **Synchronous:** the protocol proceeds in rounds. Every message that is sent arrives within a known time frame
  - **Asynchronous (eventual delivery):** the adversary can impose arbitrary (finite) delay on any message
  - **Fully Asynchronous:** the adversary has full control over the network, can even drop messages

Execution Environment

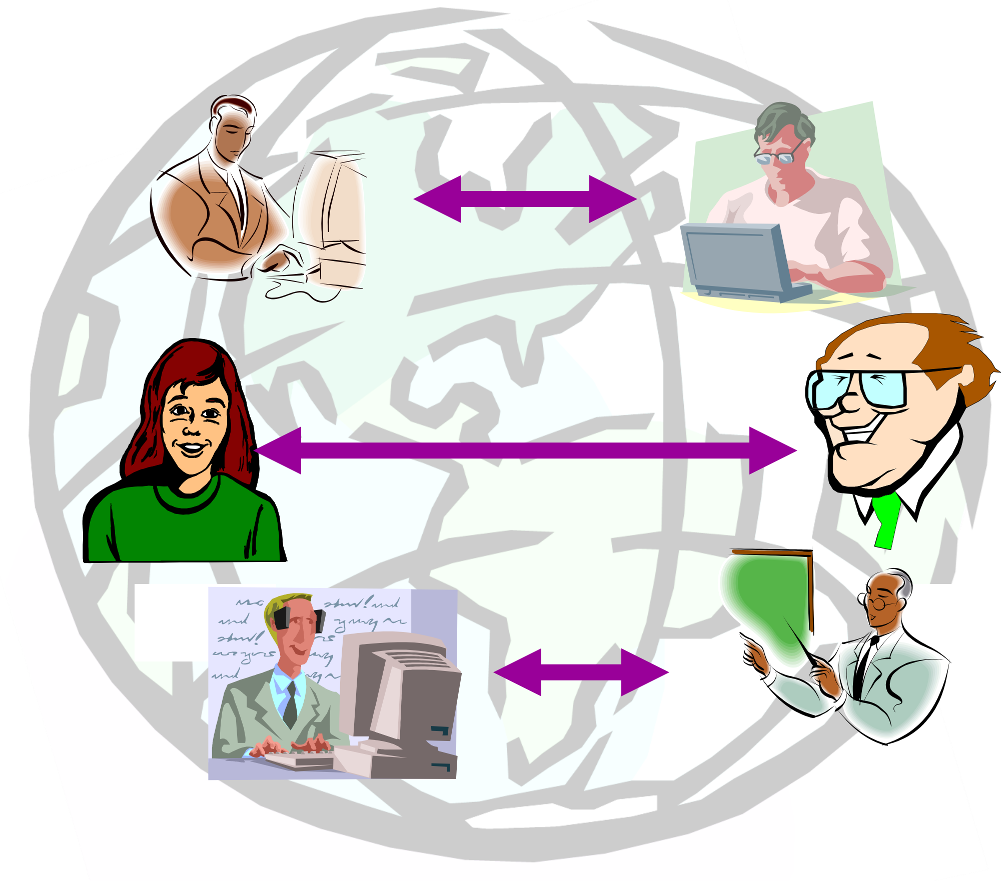
- **Stand alone**:
  - A single protocol execution at any given time (isolated from the rest of the world)
- **Concurrent general composition**:
  - Arbitrary protocols are executed concurrently
  - An Internet-like setting
  - Requires a strictly stronger definition
  - Captured by the universal composability (UC) framework

The Stand-Alone Model



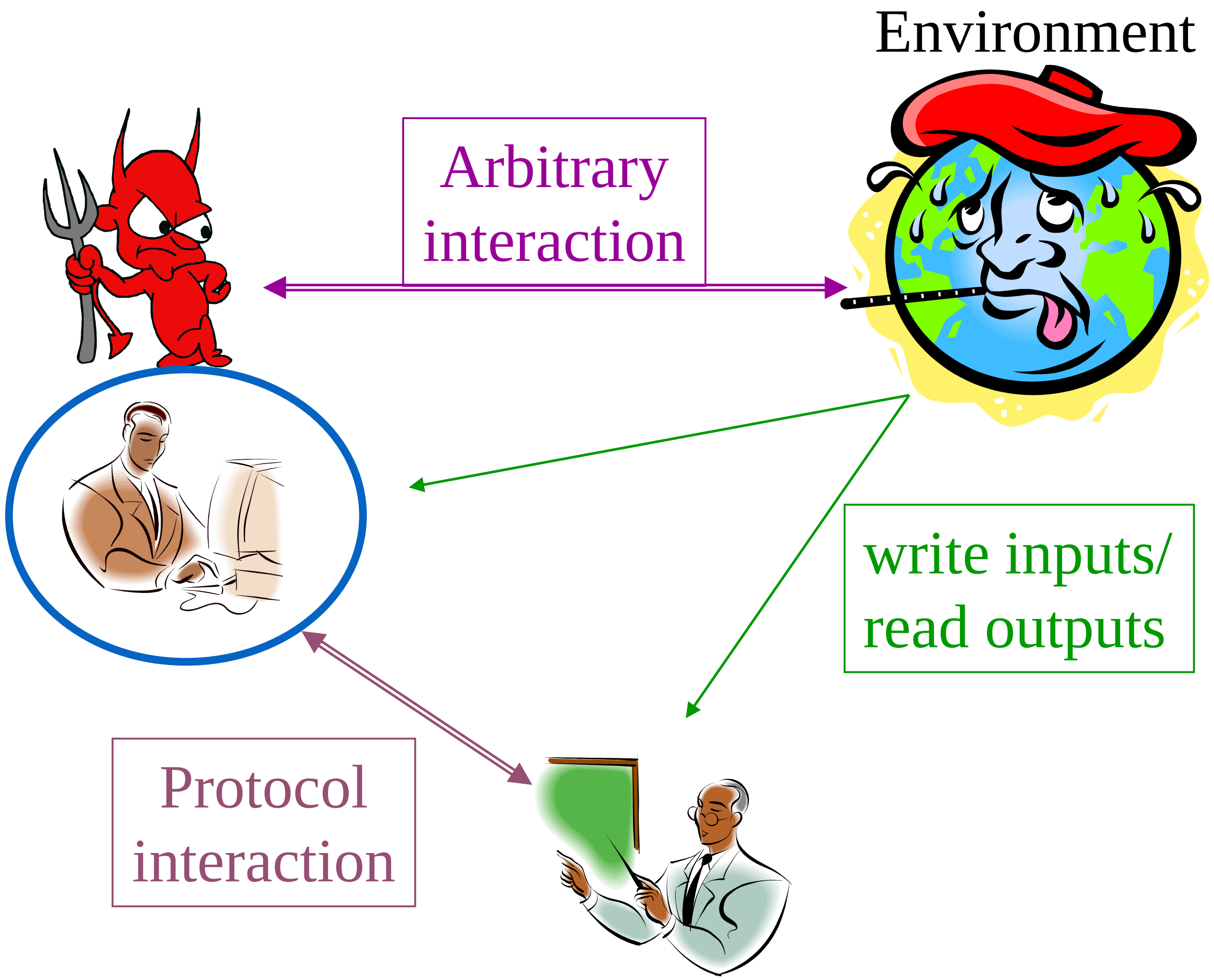
One set of parties executing a **single** protocol in **isolation**

The Concurrent Model

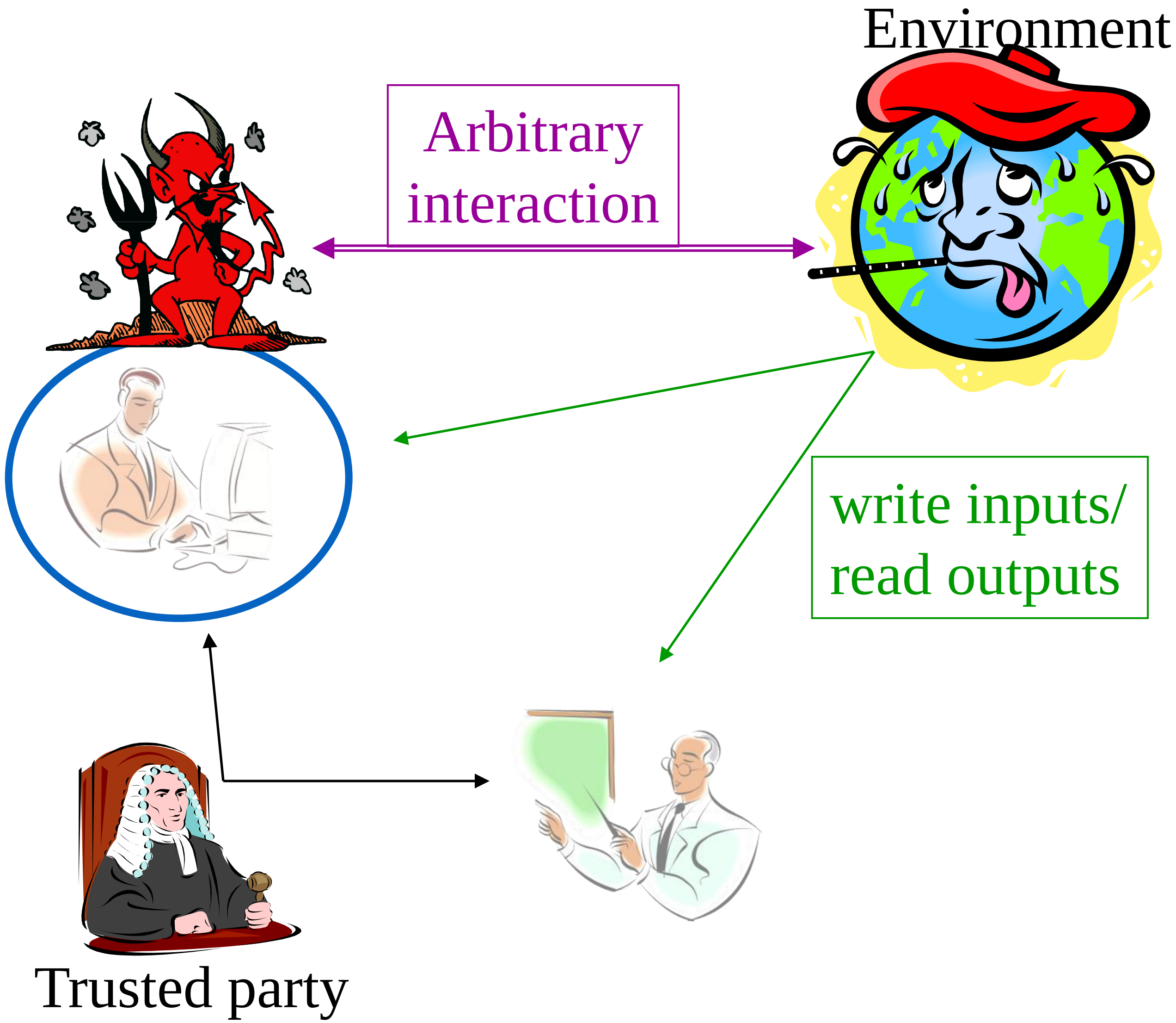


Many parties running many protocol executions

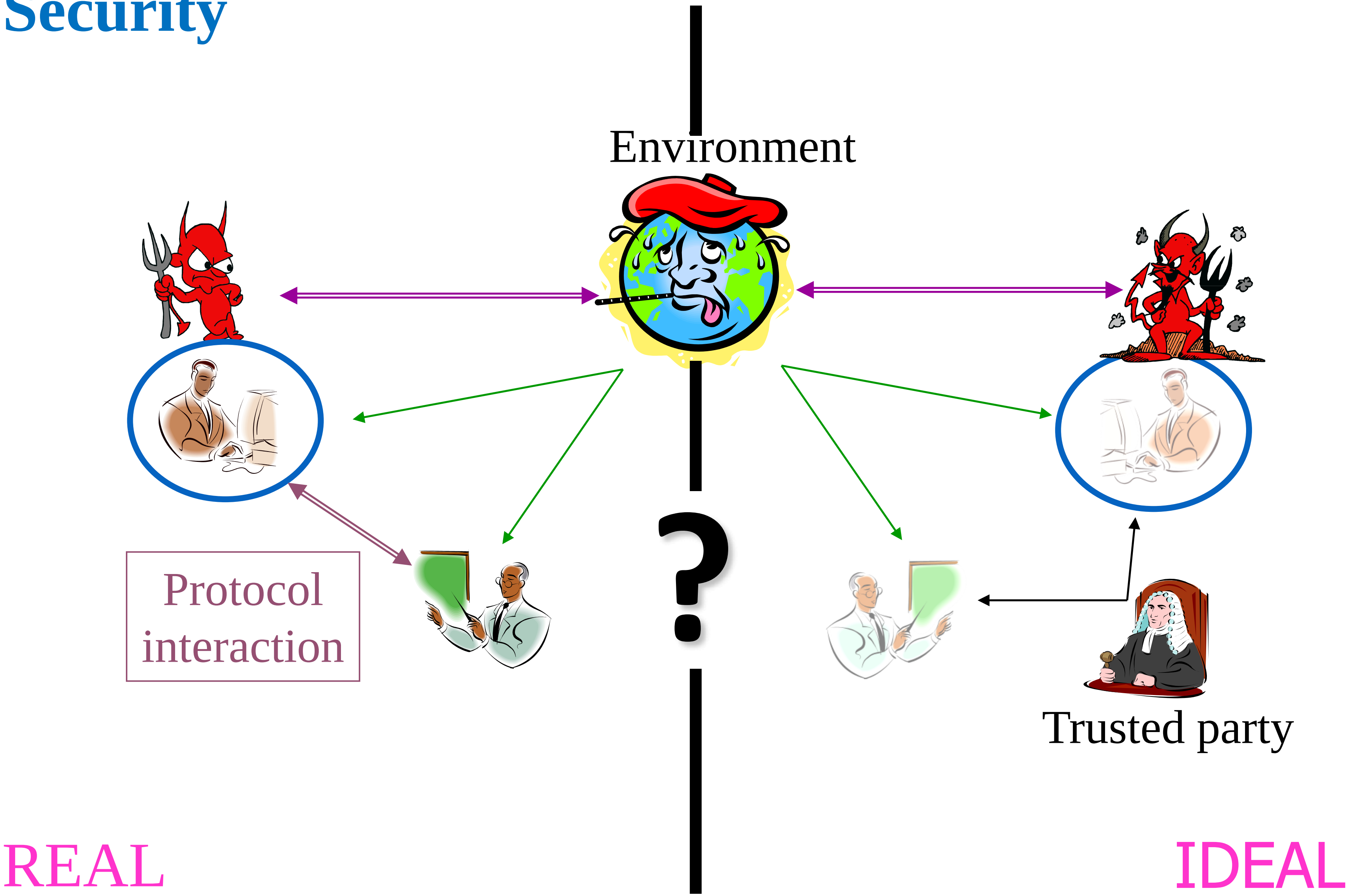
UC real model



UC ideal model



UC Security


Environment
Protocol
interaction
?
Trusted party
REAL
IDEAL

Relaxing the Definition

- Recall the ideal world (with guaranteed output delivery)
  1) Each party sends its input to the trusted party
  2) The trusted party computes $y = f(x_1, \ldots, x_n)$
  3) Trusted party sends y to each party
- This ideal world is overly ideal
- In general, fairness cannot be achieved without an honest majority
- A relaxed definition is normally considered

Security with Abort

- Ideal world without fairness and guaranteed output delivery:

a. Each party sends its input to the trusted party

b. The trusted party computes $y = f(x_1, \ldots, x_n)$

c. Trusted party sends y to the adversary

d. The adversary responds with continue/abort

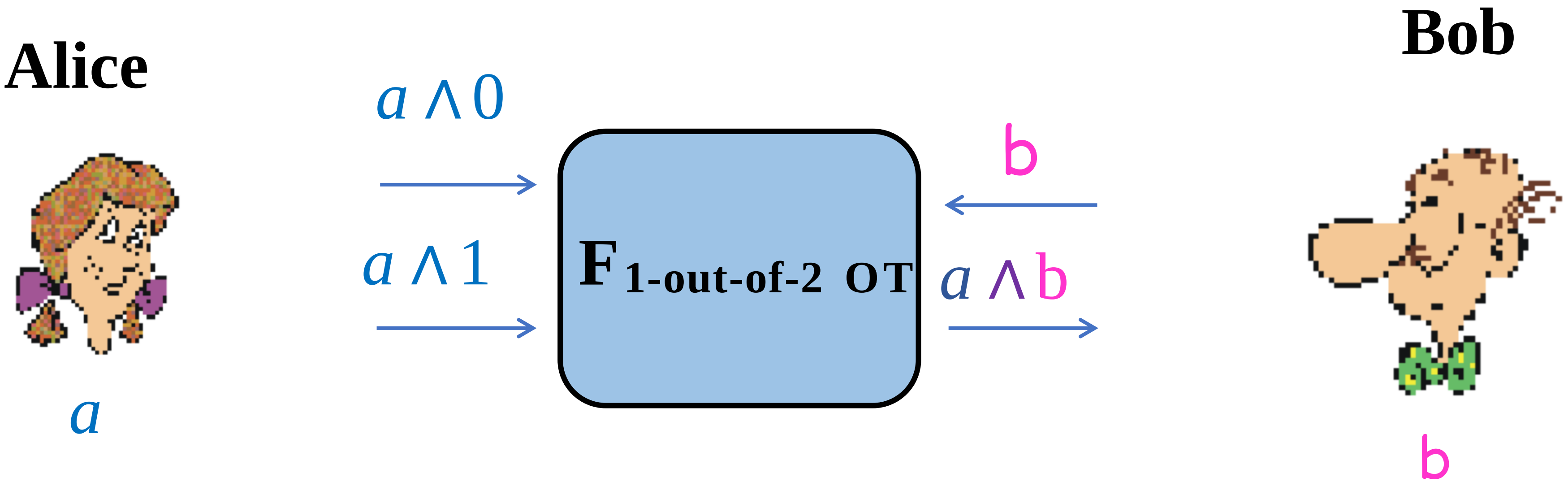
e. If continue, trusted party sends y to all parties If abort, trusted party sends $\perp$ to all parties

f. Correctness, privacy, independence of inputs are satisfied

Adversarial model

- In this lecture we consider:
  - Adversary: semi honest / malicious with static corruptions
  - Synchronous P2P network with a broadcast channel
  - Stand-alone setting
  - Probabilistic polynomial time (PPT) adversary & distinguisher (computational security)

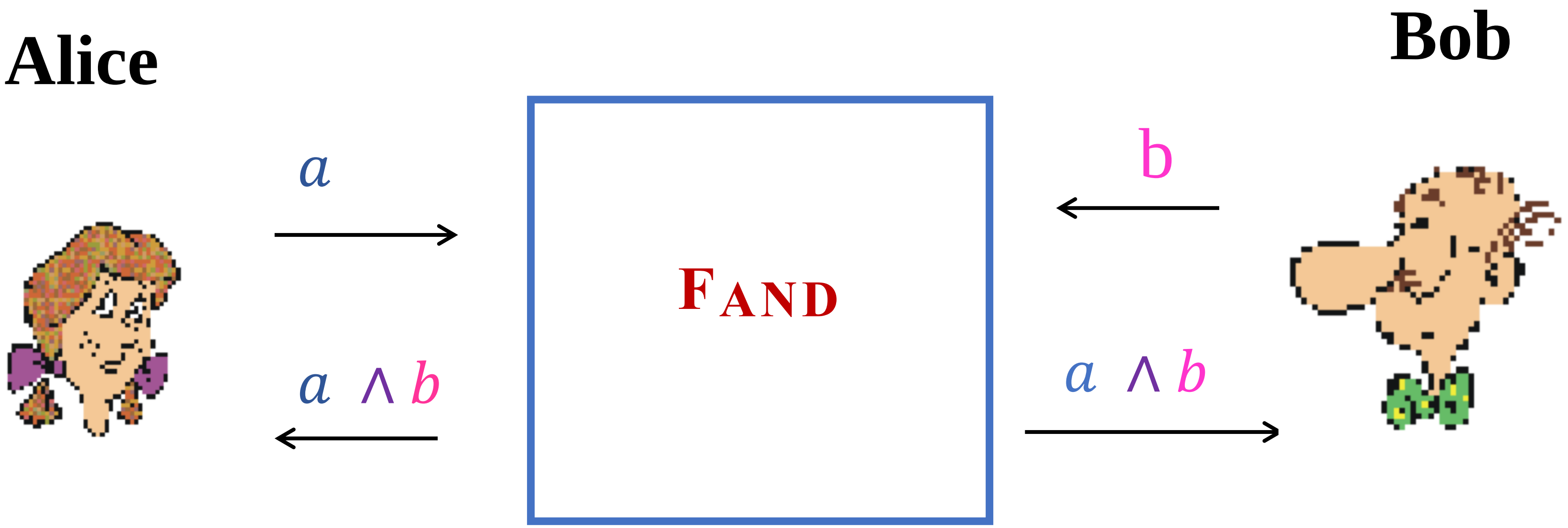
Secure AND: Π_{AND}



Bob sends $a \wedge b$ to Alice

Alice and Bob both output $a \wedge b$

Functionality



- **Theorem.** Π_{AND} is indistinguishable from F_{AND} from the perspective of an semi-honest adversary.

  - $\exists$ simulator S_1, s.t. $(View_{P_{1,real}}, Output_{P_{1,real}}) \approx (View_{P_{1,ideal}}, Output_{P_{1,ideal}})$

  - $\exists$ simulator S_2, s.t. $(View_{P_{2,real}}, Output_{P_{2,real}}) \approx (View_{P_{2,ideal}}, Output_{P_{2,ideal}})$

Semi-honest vs Malicious

- **Now to confuse you all…**
- It is clear that any protocol that is secure in the presence of malicious adversaries is secure in the presence of semi-honest adversaries
  - A malicious adversary is stronger, and can always behave semi-honestly…
- But, the simulator in the ideal model is also stronger
  - It can change its input
- Does this make a difference?

A Protocol for Binary AND: $\Pi_{x \wedge y}$

- Input: P_1 has an input bit x and P_2 has an input bit y.
- Output: The binary value $x \wedge y$ for P_2 **only**.

- The protocol:

1. P_1 sends P_2 its input bit x.
2. P_2 outputs the bit $x \wedge y$.

Semi-honest vs Malicious

Claim. $\Pi_{x \wedge y}$ securely computes the binary AND function in the presence of malicious adversaries.

Claim. $\Pi_{x \wedge y}$ does not securely compute the binary AND function in the presence of semi-honest adversaries.

Semi-honest vs Malicious

- Fixing this absurdity
  - Allow a semi-honest adversary to also change its input
  - Arguably, this is legitimate (to choose input)
  - This is called augmented semi-honest

- Theorem:
  - Security for malicious adversaries implies security for augmented semi-hones adversaries

Private set intersection (PSI)

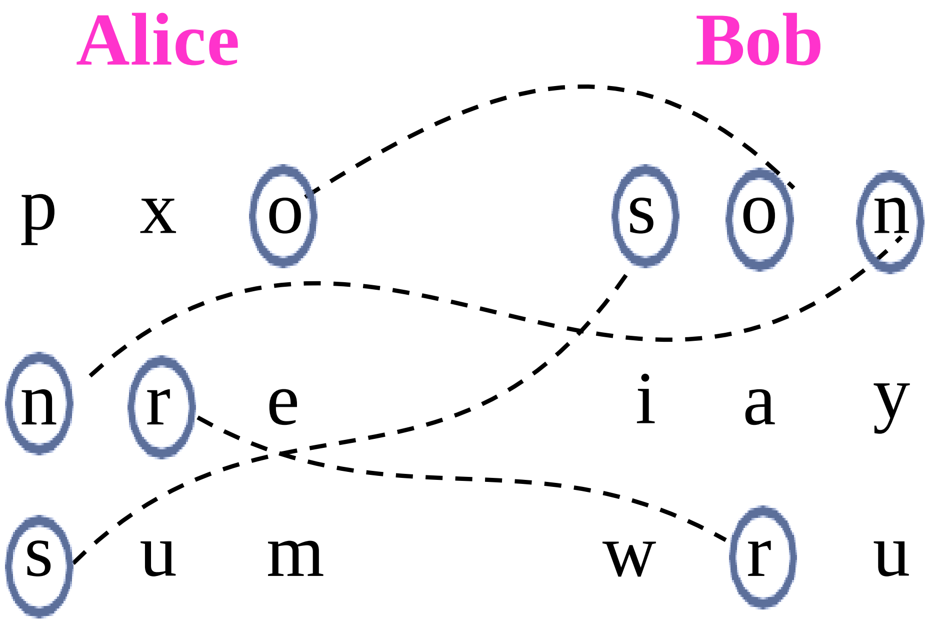
Alice

p	x	o
n	r	e
s	u	m

Bob

s	o	n
i	a	y
w	r	u

Private set intersection (PSI)



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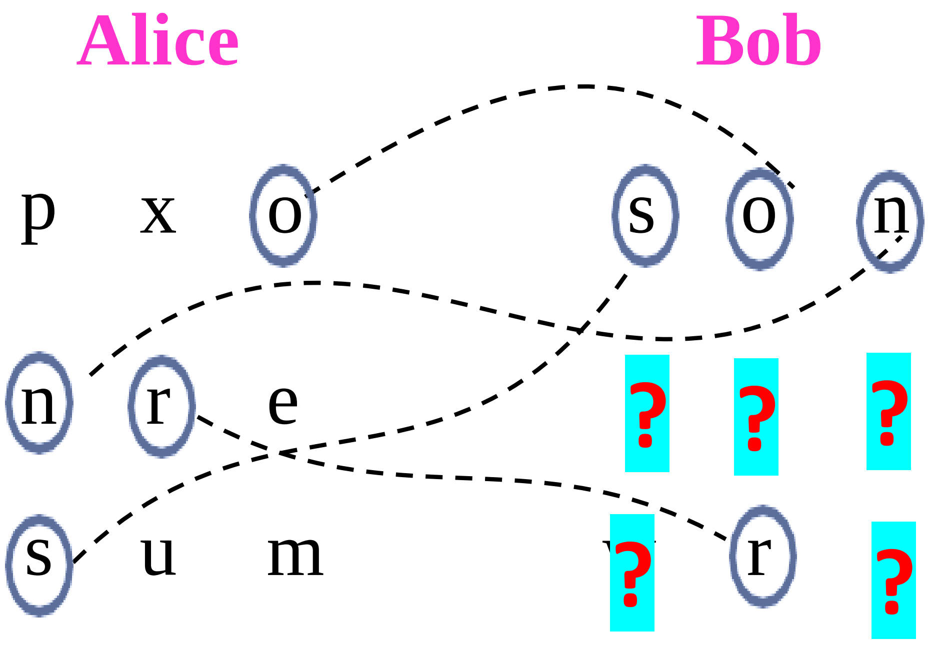

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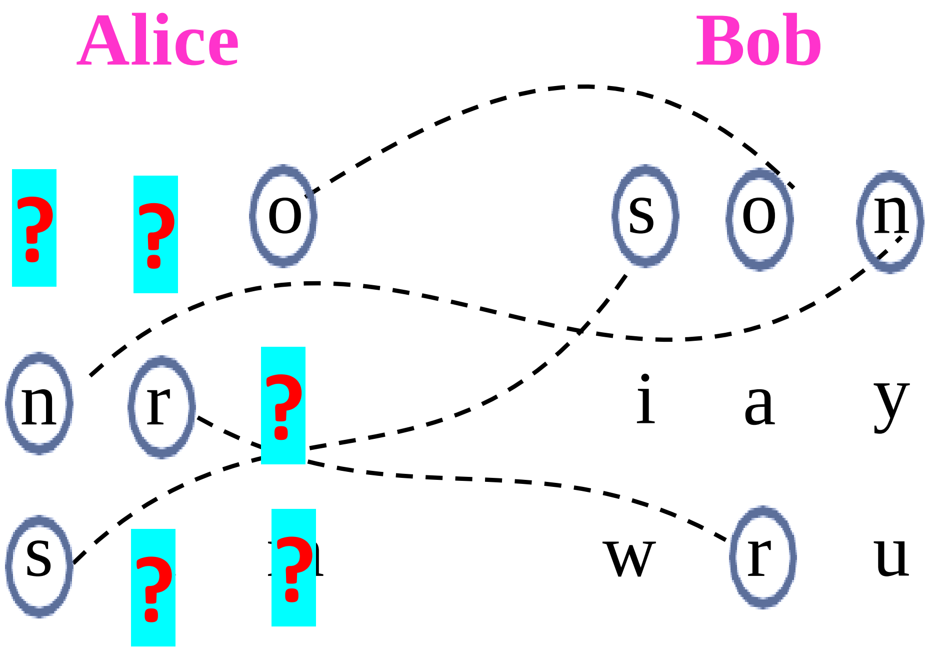

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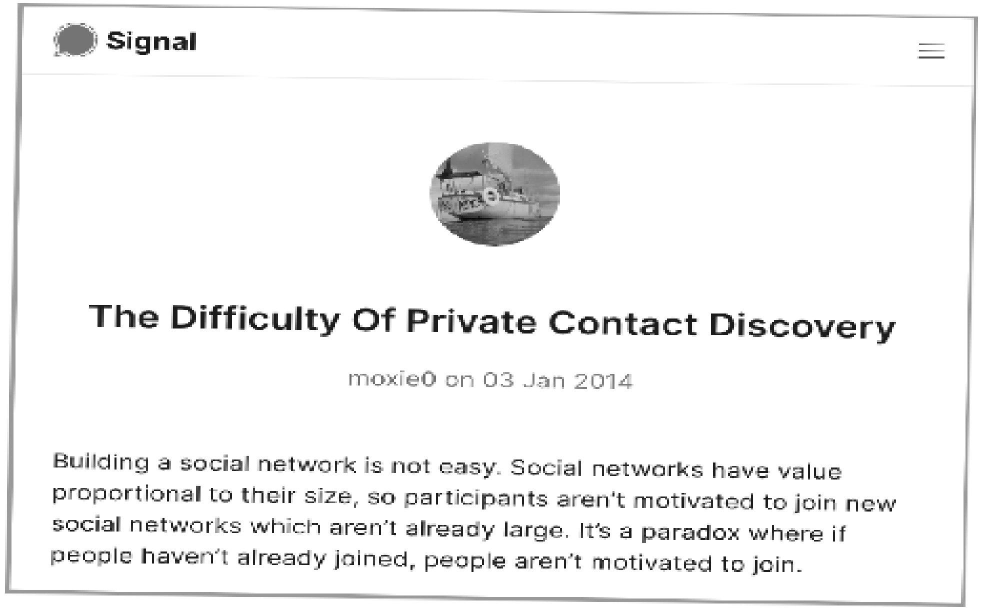

Signal

The Difficulty Of Private Contact Discovery

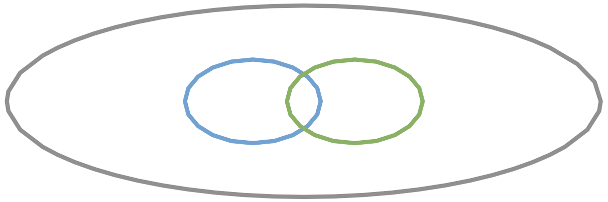
moxie0 on 03 Jan 2014

Building a social network is not easy. Social networks have value proportional to their size, so participants aren't motivated to join new social networks which aren't already large. It's a paradox where if people haven't already joined, people aren't motivated to join.

{my phonea contacts} ∩ {users of your service}

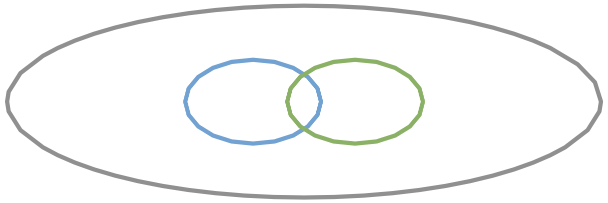
Private set intersection (PSI)

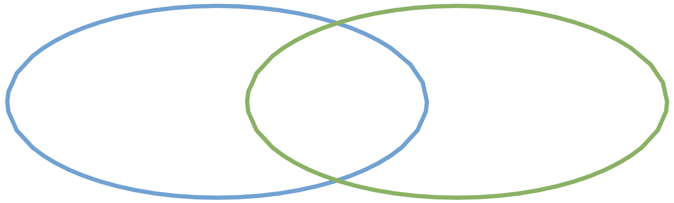
{my phonea contacts} ∩ {users of your service}



PSI on **small sets** (hundreds)

- private availability poll
- key agreement techniques





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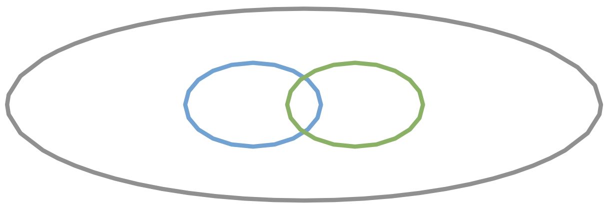
PSI on **large sets** (millions)

- double-registered voters
- OT extension; combinatorial tricks



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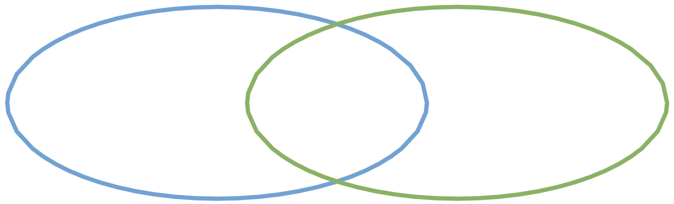

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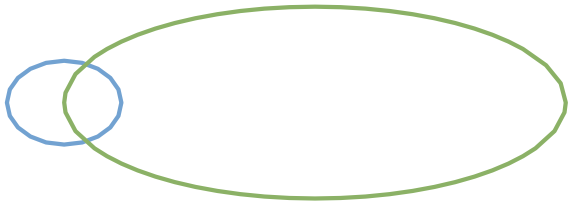
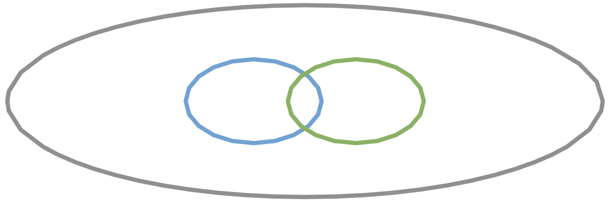


PSI on **asymmetric** sets (100 : billion)

- contact discovery; password checkup
- offiine phase; leakage



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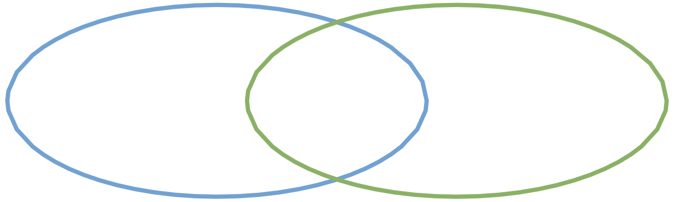

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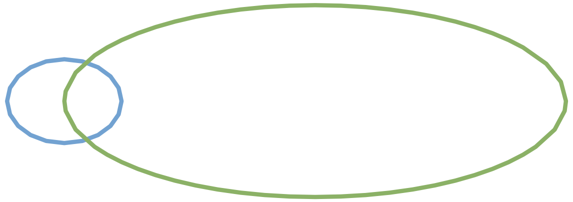

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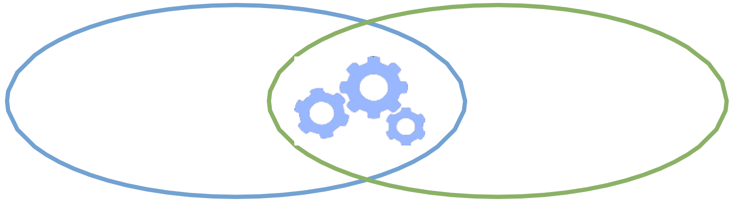


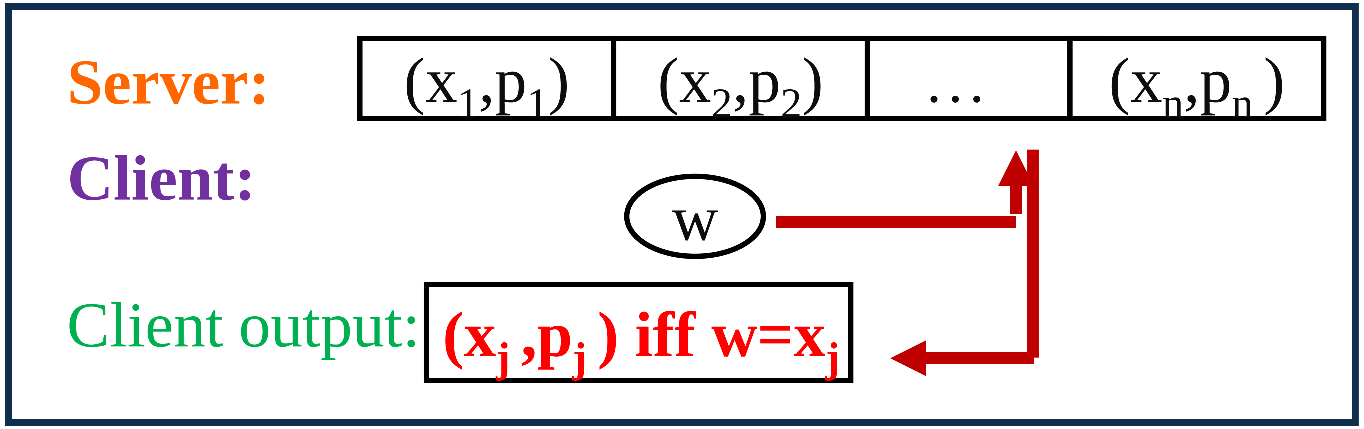
Computing on the intersection

- sales statistics about intersection
- generic secure computation

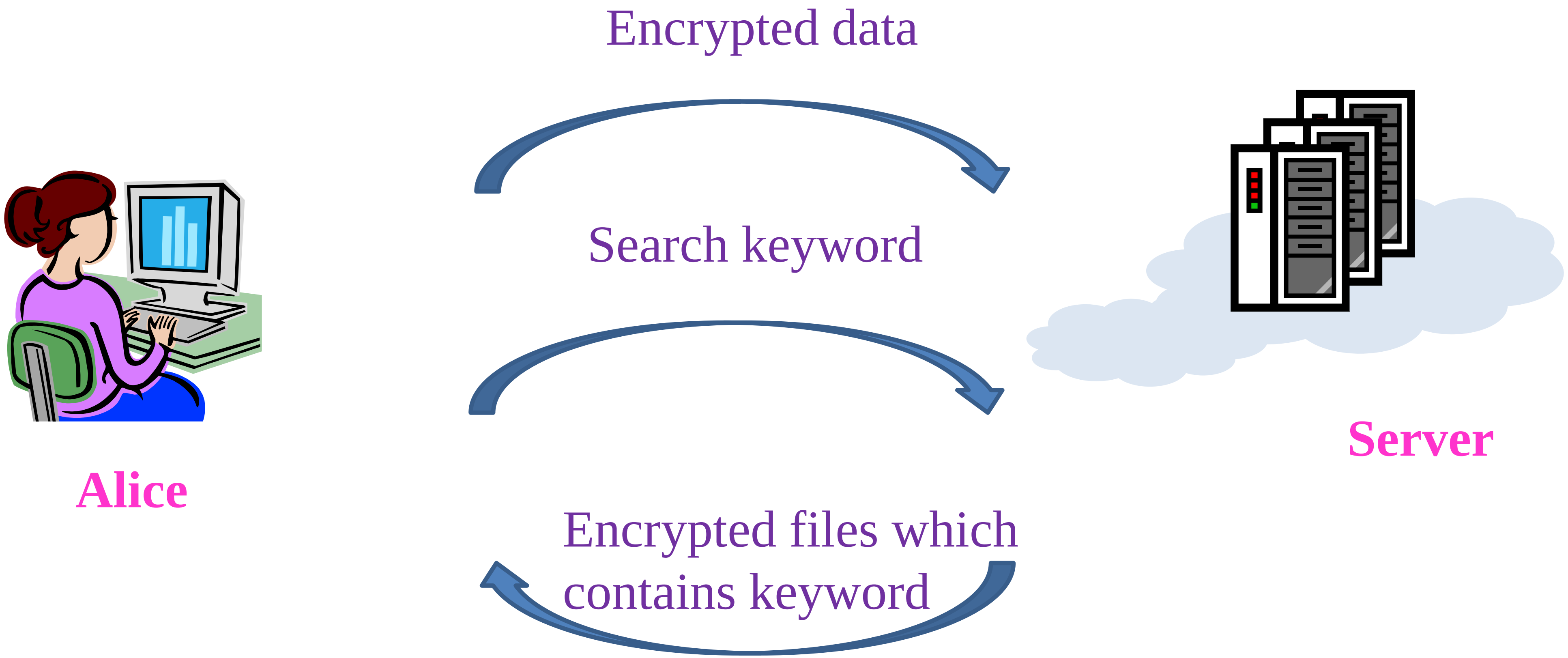
Keyword Search

- **Input:**
  - Server: database $X=\{ ((x_i,p_i)) \}$, $1 \leq i \leq N$
    - x_i is a keyword
    - p_i is the payload
  - Client: search word w

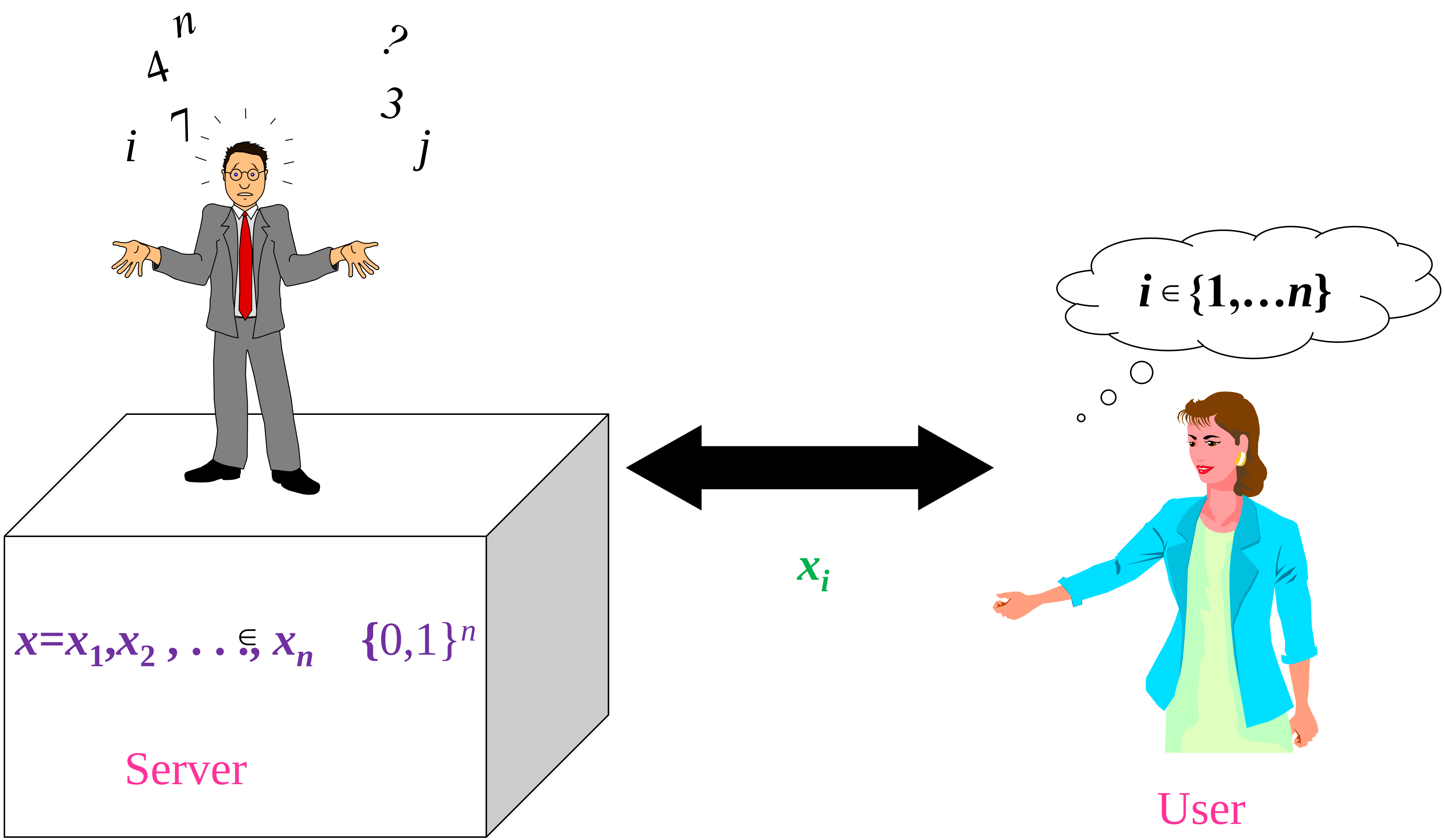
- **Output:**
  - Server: nothing
  - Client:
    - p_i if $\exists\, i : x_i = w$
    - otherwise nothing



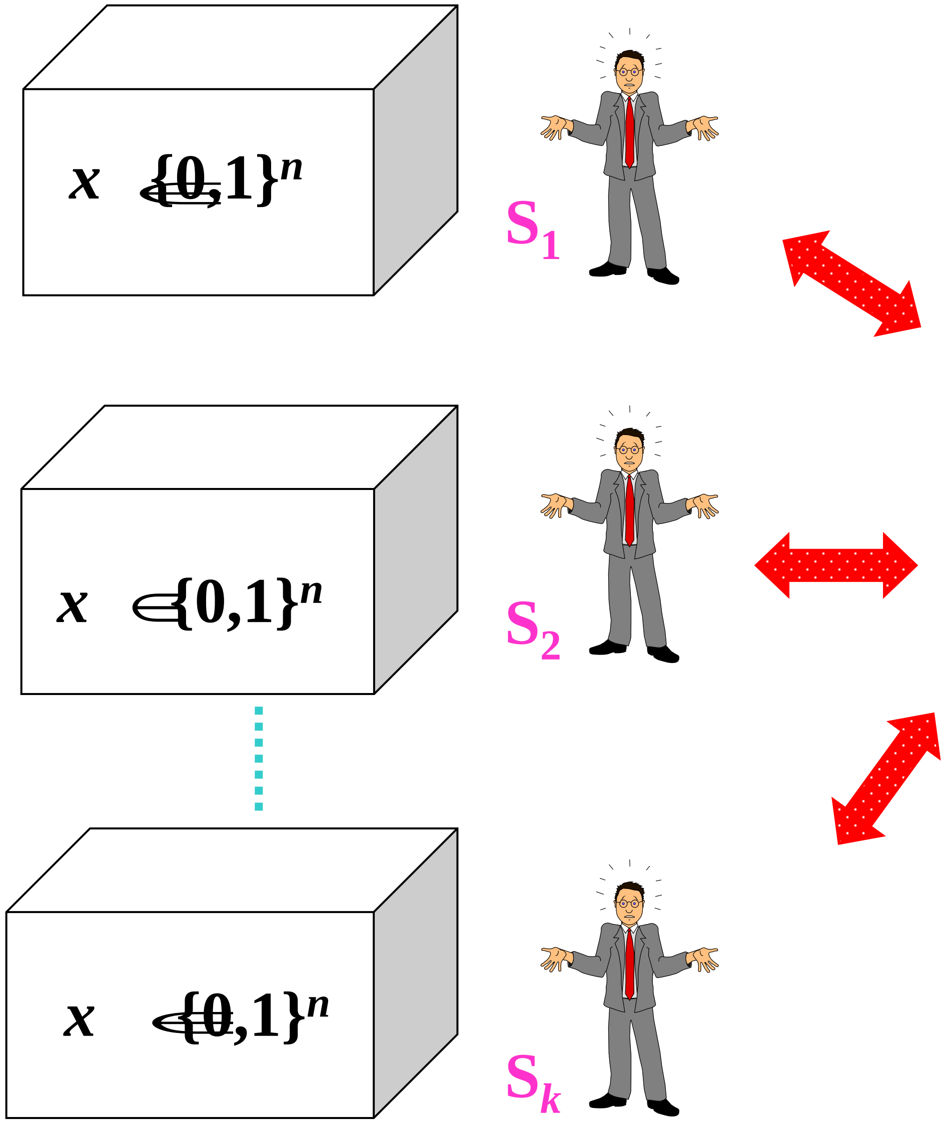
Searchable Encryption

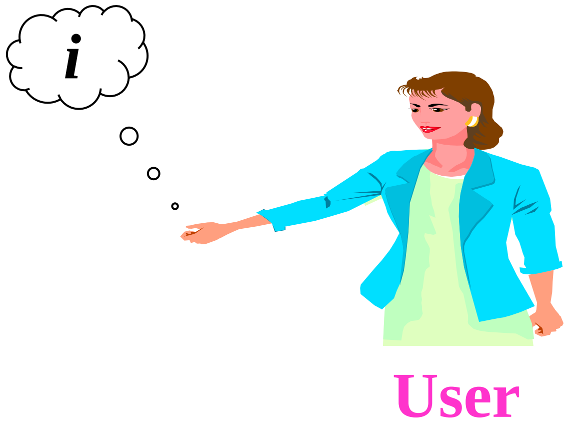


Private Information Retrieval (PIR)



k-Server PIR



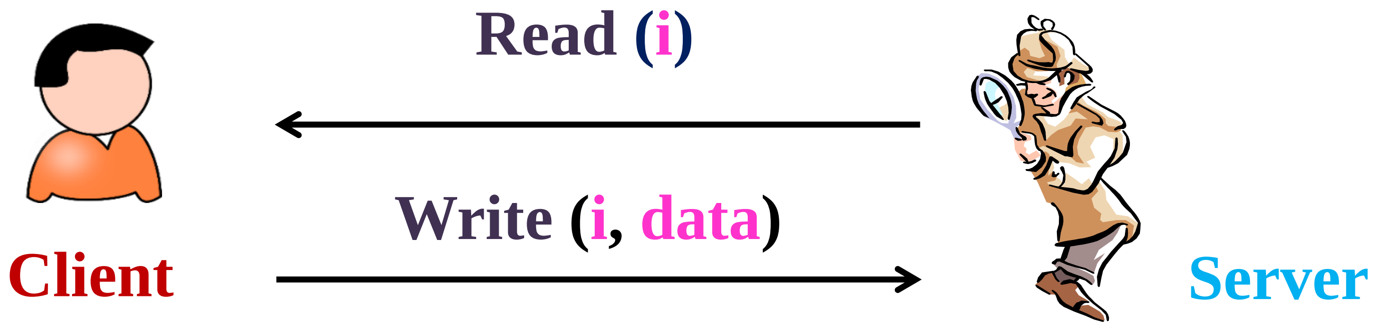


Correctness: User obtains x_i

Privacy: No *single* server gets information about i

Oblivious Random Access Machine (ORAM)

- A machine is **oblivious** if its **sequence of accessing (memory) locations is indistinguishable** for any two inputs with the same length.



- The **server** cannot gain any information from the access pattern of **client**'s Read/ Write requests.